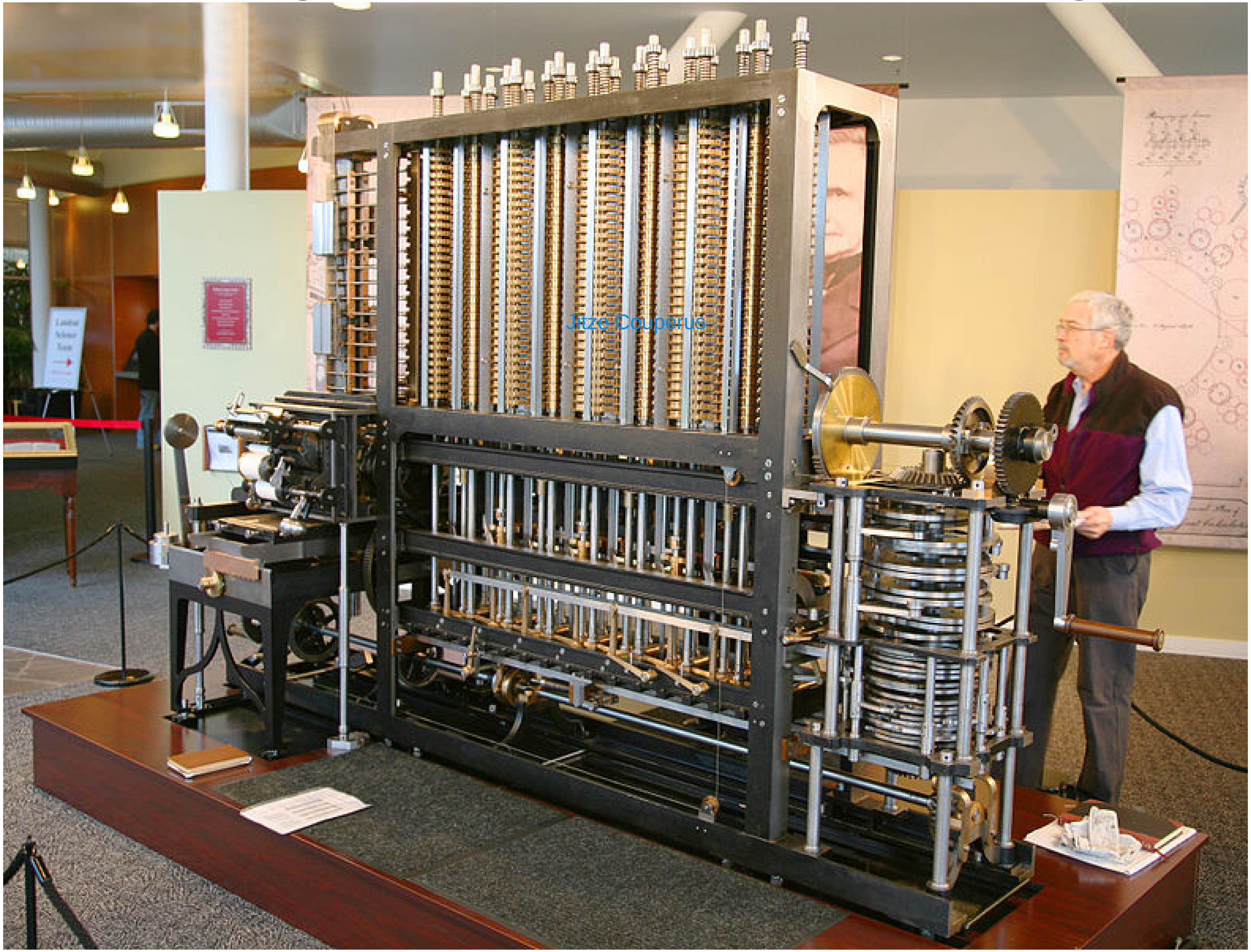
ENGR-354 Digital Logic
Intro to logic circuits and boolean algebra



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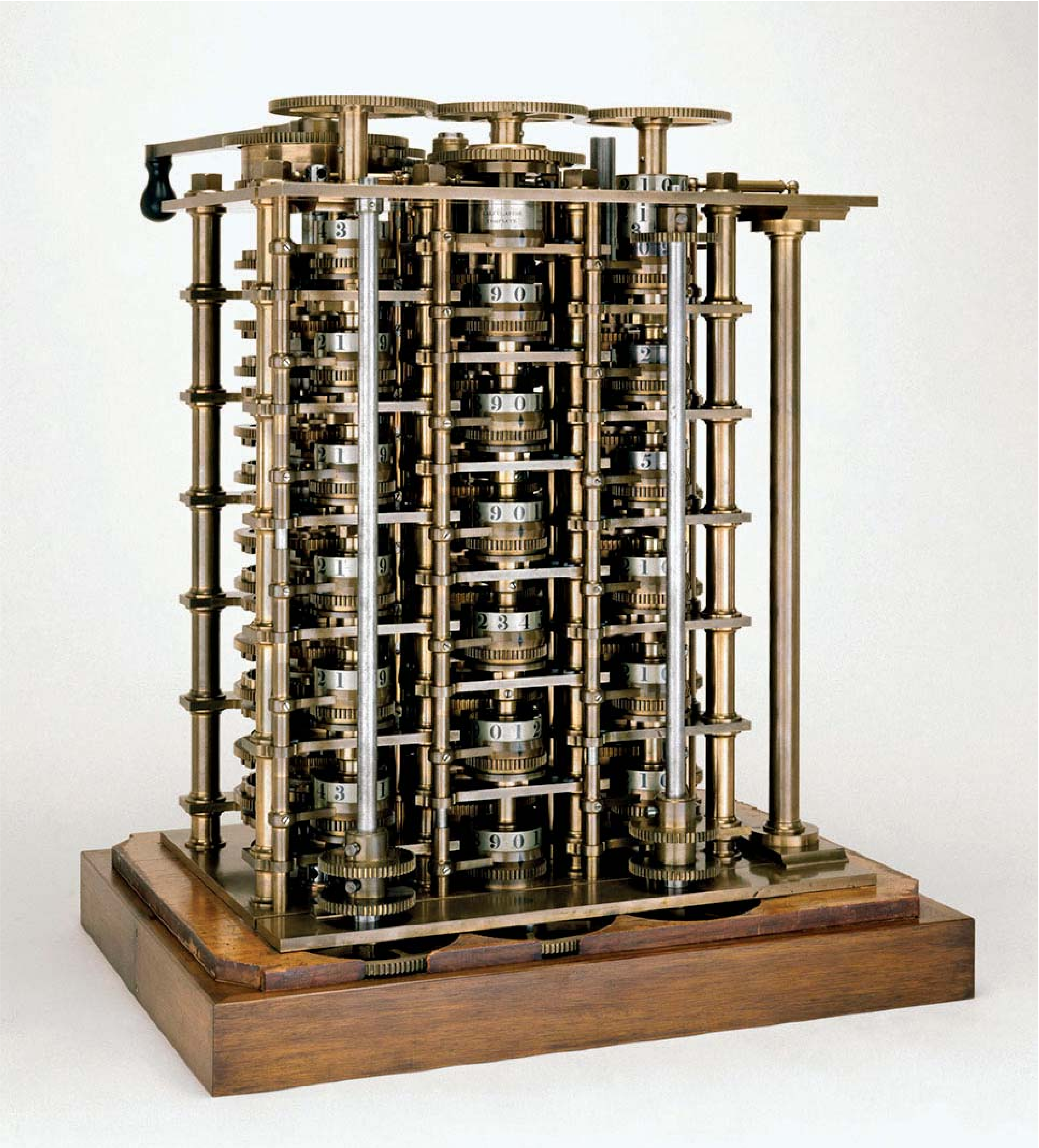

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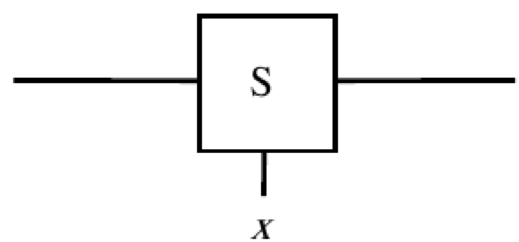
Binary Logic Circuits

- Logic circuits perform operations on digital signals;
- These circuits are implemented using electronic components;
- Binary logic circuits can be found in one of two states
  - 0 or 1;
  - off or on;
  - down or up;
  - not asserted or asserted;
  - etc.

Switch Representation

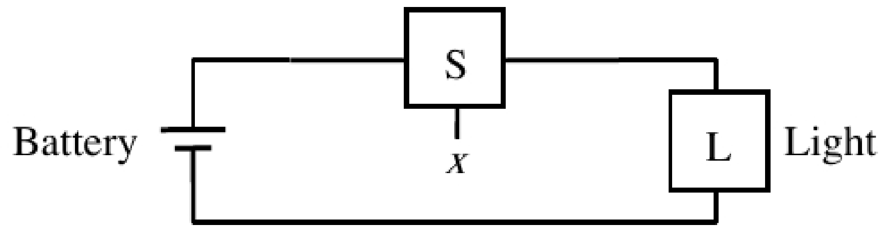
(a) Two states of a switch



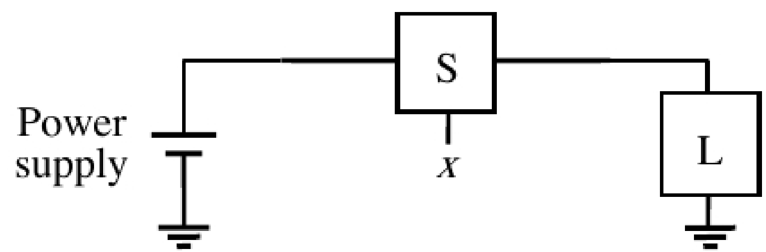
(b) Symbol for a switch

Switch Example

$$L(x) = x$$

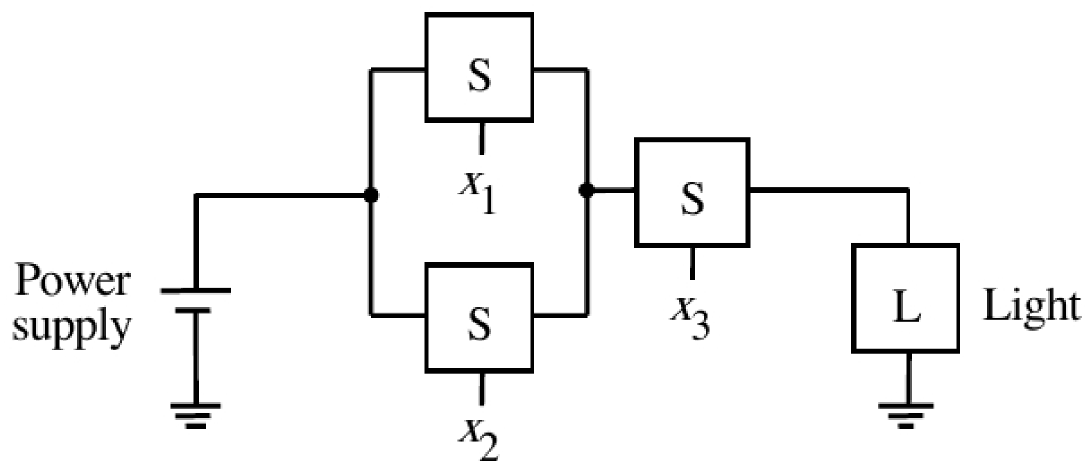


(a) Simple connection to a battery



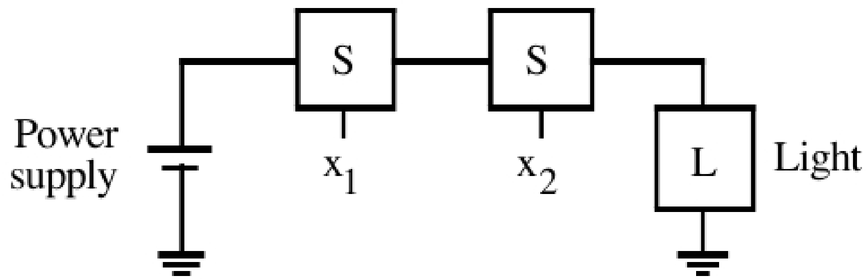
(b) Using a ground connection as the return path

A Series-Parallel Example



$$L(x_1, x_2, x_3) = (x_1 + x_2) \cdot x_3$$

Two Basic Functions

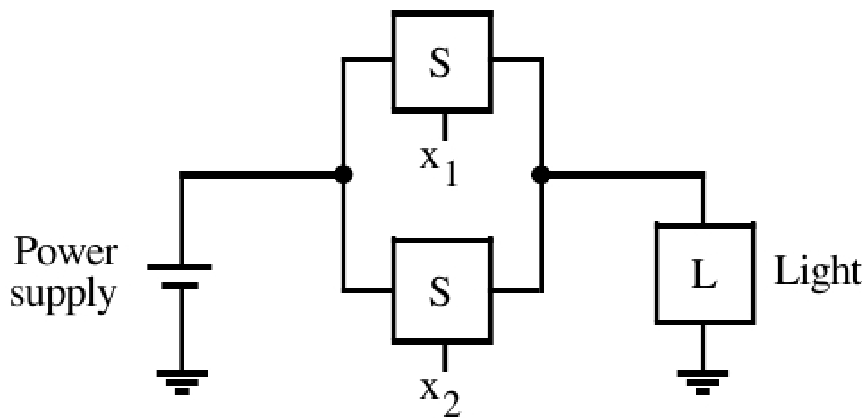


(a) The logical AND function (series connection)

$L(x_1, x_2) = x_1 \cdot x_2$

$L = 1$ if $x_1 = 1$ **and** $x_2 = 1$

$L = 0$ otherwise



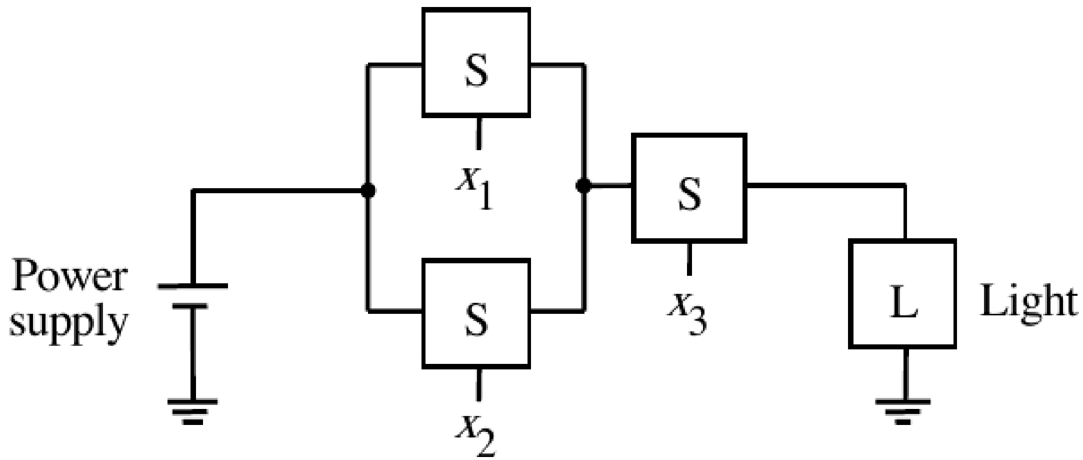
(b) The logical OR function (parallel connection)

$L(x_1, x_2) = x_1 + x_2$

$L = 1$ if $x_1 = 1$ **or** $x_2 = 1$

$L = 0$ otherwise

A Series-Parallel Example



$$L(x_1, x_2, x_3) = (x_1 + x_2) \cdot x_3$$

Truth Tables

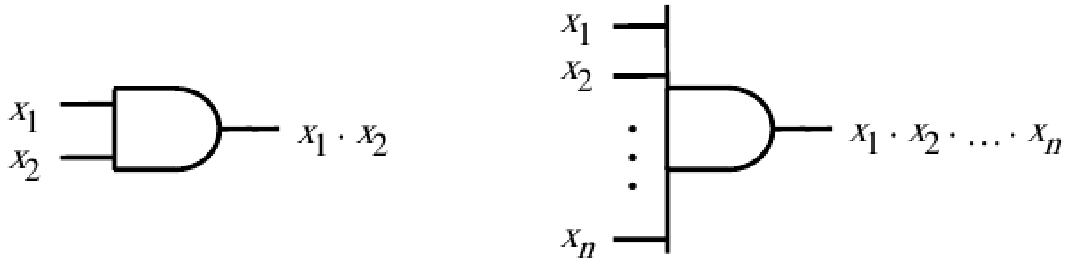
- All combinations of inputs on the left;
- Outputs on the right;
- 2-input **AND** and **OR** functions shown below.

x_1	x_2	$x_1 \cdot x_2$	$x_1 + x_2$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1
		AND	OR

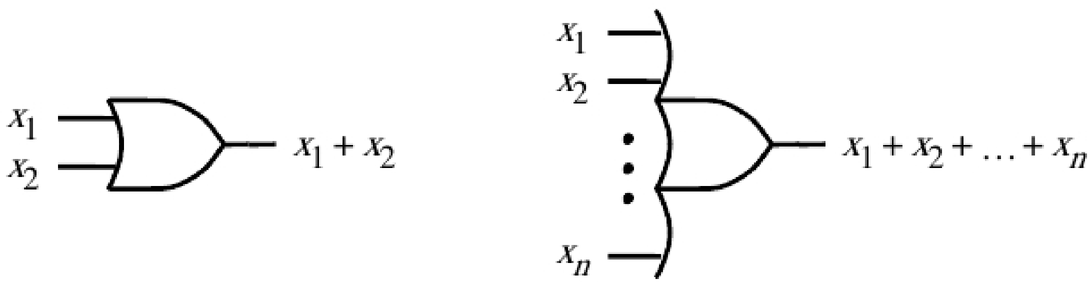
3-Input And and Or Functions

x_1	x_2	x_3	$x_1 \cdot x_2 \cdot x_3$	$x_1 + x_2 + x_3$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

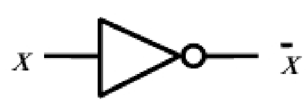
Basic Gates



(a) AND gates

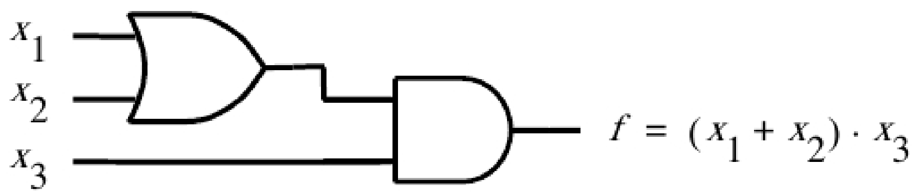


(b) OR gates

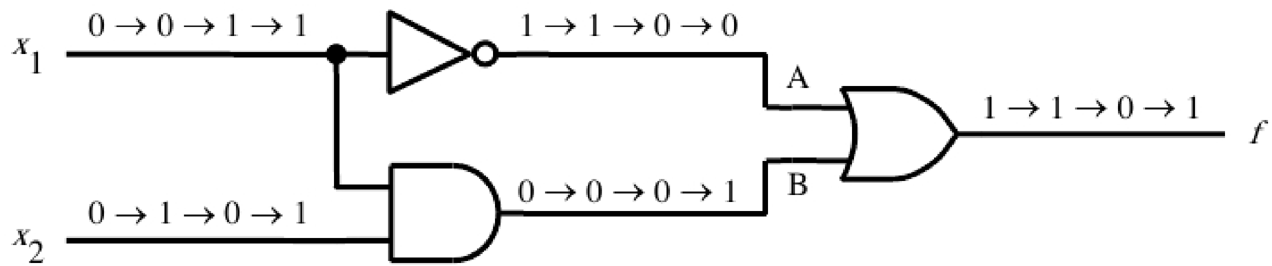


(c) NOT gate

Example Using Basic Gates



Sequencing of Inputs

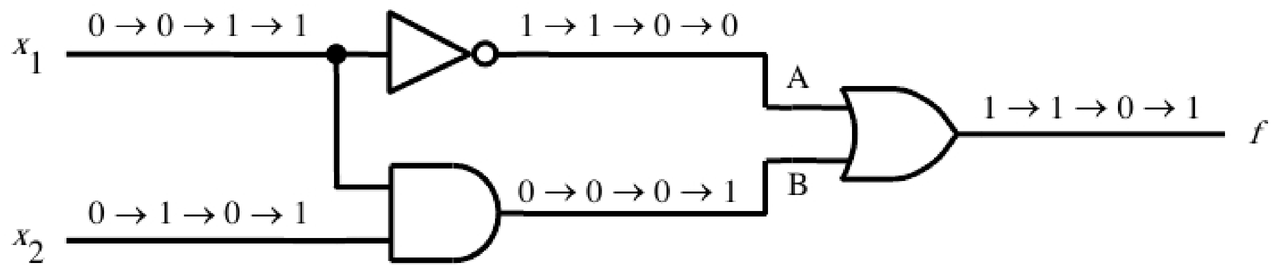


Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

Truth table for f

Sequencing of Inputs



Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$

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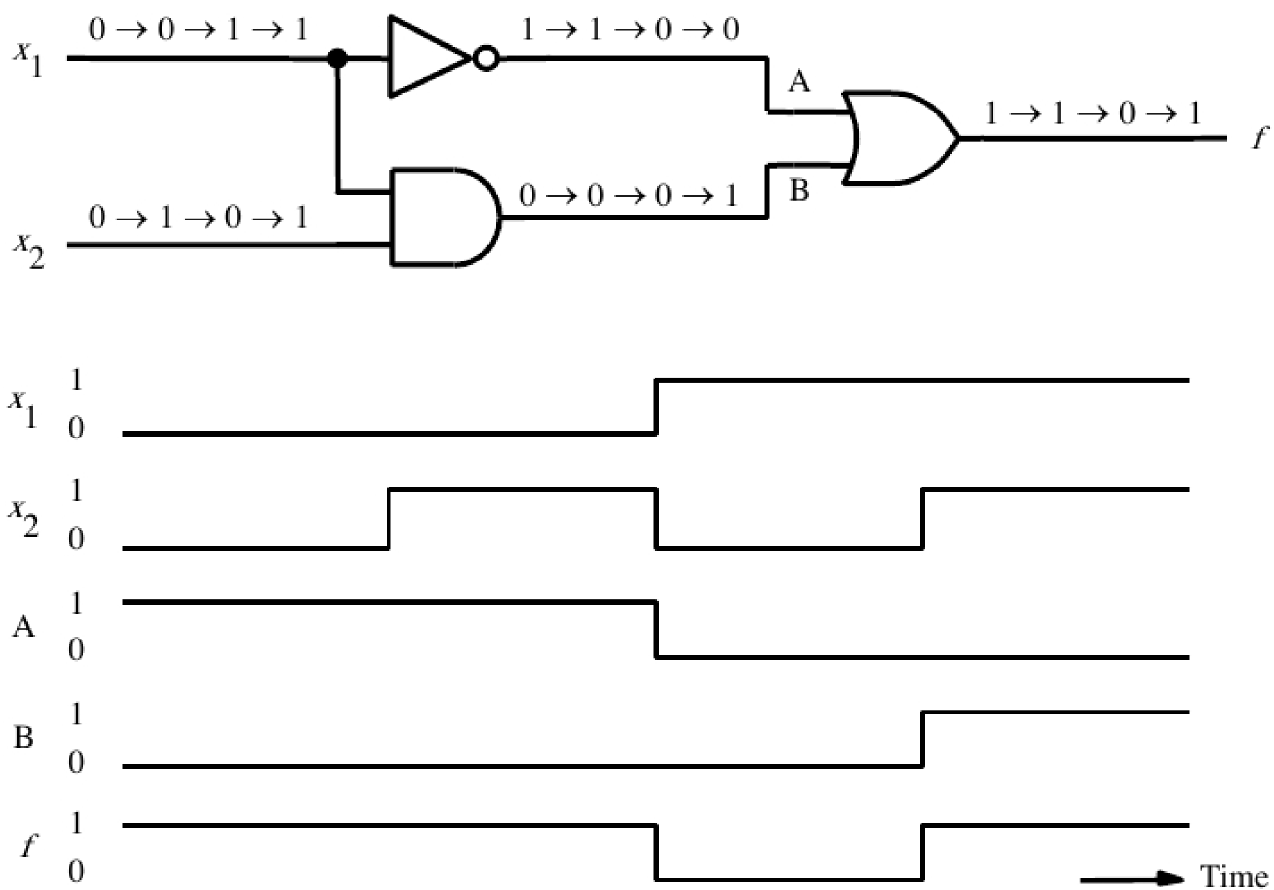
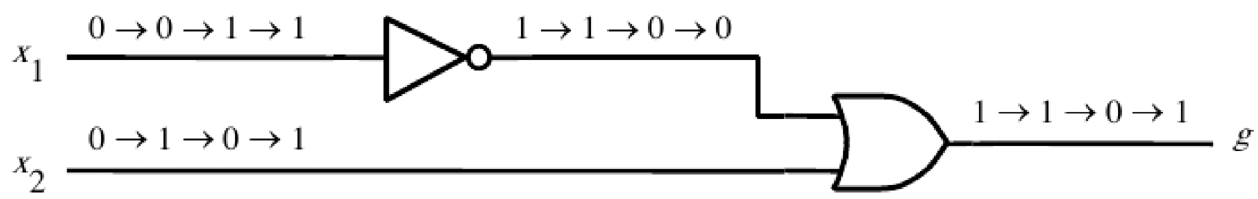
Truth table for f

Timing Diagram



Timing diagram

Example



Network that implements $g = \bar{x}_1 + x_2$

- Draw a timing diagram below:

Boolean Algebra

- In 1849, George Boole published a scheme for describing logical thought and reasoning;
- In 1930s, Claude Shannon applied Boolean algebra to describe circuits built with switches;
- Boolean algebra provides the mathematical foundation for digital design.

Notation

- INVERSION: $\overline{x} = x' = !x = \text{NOT } x$

  $\overline{f(x_1,x_2)} = \overline{x_1 + x}_2 = (x_1 + x_2)' = !(x_1 + x_2)$

  $= \text{NOT}(x_1 + x_2)$
- AND: $x_1 \cdot x_2 = x_1 \wedge x_2 = x_1\, x_2$
- OR: $x_1 + x_2 = x_1 \vee x_2$

Precedence of Operations

- In the absence of parentheses, operations are performed in this order: NOT, AND, OR

$$x_1 x_2 + x_1' x_2' = (x_1 x_2) + ((x_1') (x_2'))$$

Principle of Duality

- On the following pages, axioms and theorems are listed in pairs to show the principle of duality;
- Given a logic expression, its *dual* is found by exchanging + and • operators and 0's and 1's;
- The dual of any true statement is **always** true.

Axioms of Boolean Algebra

1. $0 \cdot 0 = 0$ $\quad$ $1 + 1 = 1$
2. $1 \cdot 1 = 1$ $\quad$ $0 + 0 = 0$
3. $0 \cdot 1 = 1 \cdot 0 = 0$ $\quad$ $1 + 0 = 0 + 1 = 1$
4. if $x = 0$ then $\overline{x} = 1$ $\quad$ if $x = 1$ then $\overline{x} = 0$

Single-Variable Theorems

5. $x \cdot 0 = 0$ $\qquad x + 1 = 1$
6. $x \cdot 1 = x$ $\qquad x + 0 = x$
7. $x \cdot x = x$ $\qquad x + x = x$
8. $x \cdot \overline{x} = 0$ $\qquad x + \overline{x} = 1$
9. $\overline{\overline{x}} = x$

2- and 3-Variable Properties

10a. $x \cdot y = y \cdot x$	Commutative
10b. $x + y = y + x$	
11a. $x \cdot (y \cdot z) = (x \cdot y) \cdot z$	Associative
11b. $x + (y + z) = (x + y) + z$	
12a. $x \cdot (y + z) = x \cdot y + x \cdot z$	Distributive
12b. $x + y \cdot z = (x + y) \cdot (x + z)$	

2- and 3-Variable Properties

13a. $x + x \cdot y = x$	Absorption
13b. $x \cdot (x + y) = x$	
14a. $x \cdot y + x \cdot \overline{y} = x$	Combining
14b. $(x + y) \cdot (x + \overline{y}) = x$	
15a. $\overline{x \cdot y} = \overline{x} + \overline{y}$	DeMorgan's Thm
15b. $\overline{x + y} = \overline{x} \cdot \overline{y}$	
16. $x + \overline{x} \cdot y = x + y$	$x \cdot (\overline{x} + y) = x \cdot y$

Truth Table Proof of DeMorgan's Theorem

15a. $\overline{x \cdot y} = \overline{x} + \overline{y}$ DeMorgan's Theorem

x	y	$x \cdot y$	$\overline{x \cdot y}$	$\overline{x}$	$\overline{y}$	$\overline{x} + \overline{y}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

LHS: $x \cdot y$, $\overline{x \cdot y}$ — RHS: $\overline{x}$, $\overline{y}$, $\overline{x} + \overline{y}$

BOOLAN ALGEBRA EXAMPLES

1) $(A+B)(\bar{A}+\bar{B}) =$

2) $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C =$