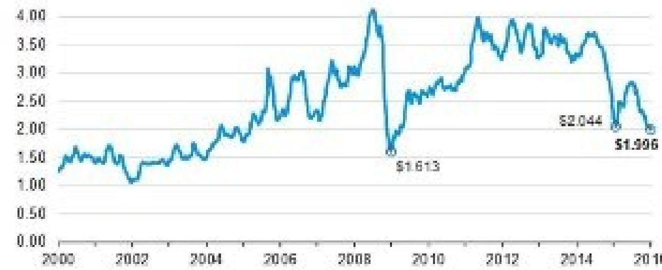
Time domain, frequency domain

- We usually think of time as the natural domain for our signals. Consider:
  - waveforms on an oscilloscope
  - your GPA charted over four years
  - the price of gasoline over time



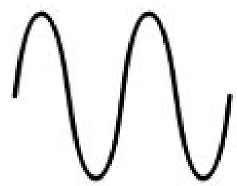
- Sometimes it is more convenient to think in terms of frequency:
  - audio equalizer
  - transfer function of a mechanical system—IDing frequencies of modes, etc.
- These are two views of a <u>single, underlying reality</u>.
  For example, a transfer function and an impulse response are equally valid ways of describing the *same* system behavior.

Time domain, frequency domain

- From a mathematical point of view, describing a signal as a function of time or as a function of frequency are equally valid.

- It is exactly the same as in linear algebra when you learned to express vectors as sums of basis vectors—many bases are possible.

- Frequency domain:
  - basis is a set of sinusoids
  - express any signal as a sum of sinusoids



- We can find these sinusoids using Fourier analysis.

Periodic signals

A signal $y(t)$ is *periodic* if and only if

$$y(t) = y(t + T)$$

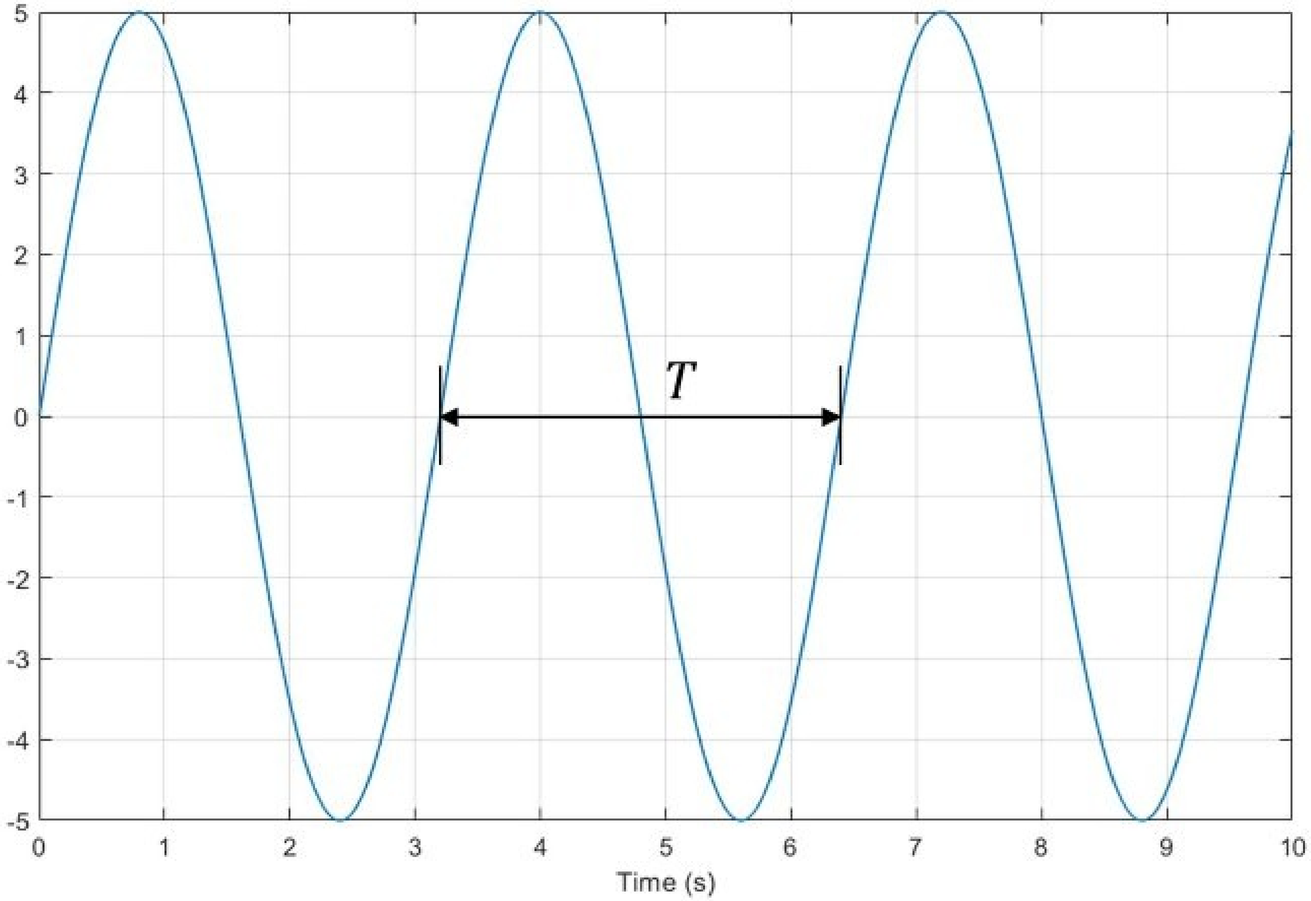
for some time shift T, called the *period*.

In other words, the signal repeats every T time units, infinitely, in both directions.

The best-known periodic signals are *sinusoids* (sines and cosines).

We will see that every periodic signal can be expressed as a sum of sines and cosines, which we call a *Fourier series*.

Sinusoid parameters: $A \sin(\omega t)$



$5 \sin\left(\frac{2\pi t}{3.2}\right)$

amplitude:
$A = 5$

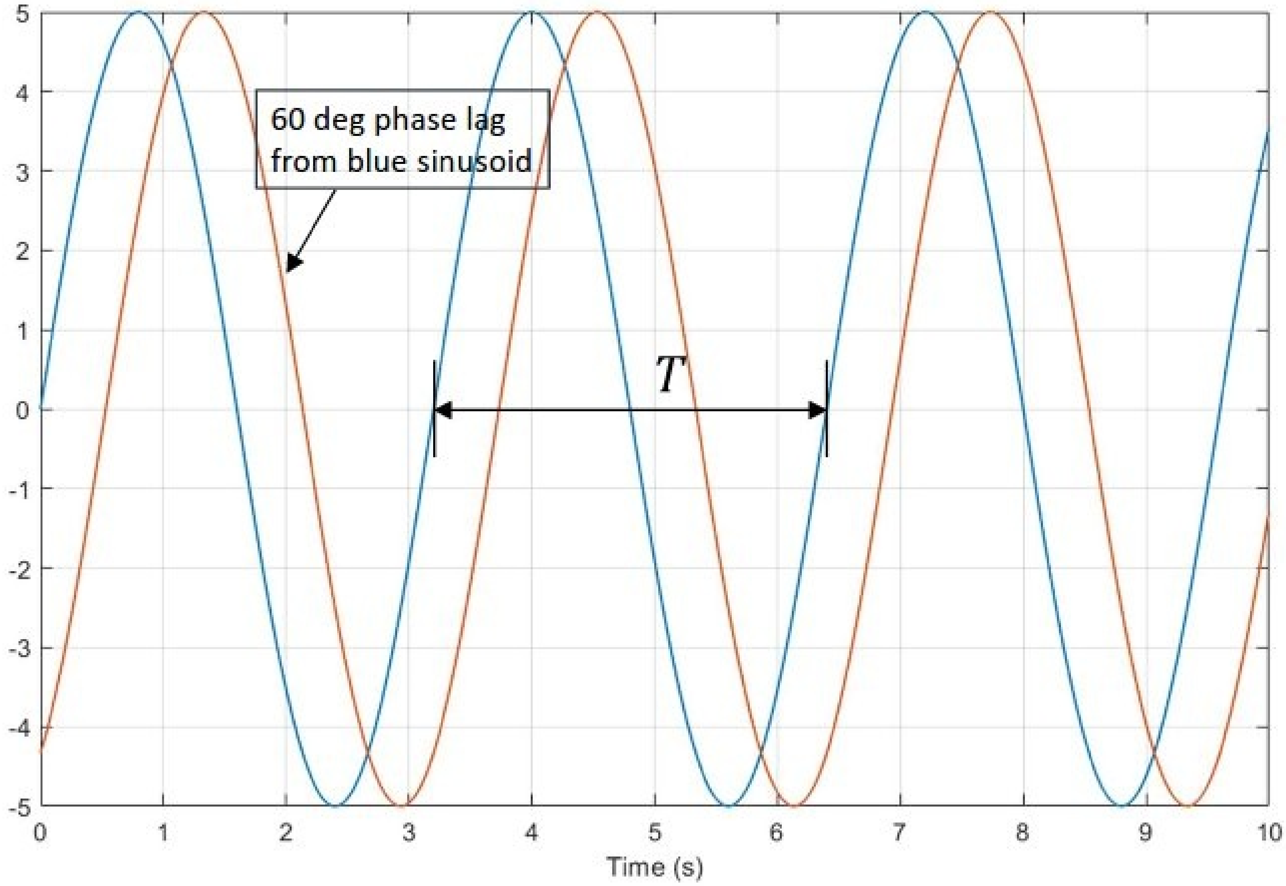
angular (circular) freq:
$\omega = \frac{2\pi}{3.2}$ rad/s

frequency:
$f = \frac{\omega}{2\pi} = \frac{1}{3.2}$ Hz

period:
$T = \frac{1}{f} = 3.2$ s

Sinusoid parameters: $A\sin(\omega t - \phi)$

$5\sin\left(\frac{2\pi t}{3.2} - \frac{\pi}{3}\right)$



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period:
$T = \frac{1}{f} = 3.2$ s

phase shift:
$\phi = \frac{\pi}{3} = 60°$ (lagging)

Fourier series

- What does this mean??! (Breathe.)
- Every periodic signal can be thought of as
  - DC component (A_0, simple average over T)
  - cosines at fundamental and harmonics: $A_1 \cos(\omega t) + A_2 \cos(2\omega t) + \cdots$
  - sines at fundamental and harmonics: $B_1 \sin(\omega t) + B_2 \sin(2\omega t) + \cdots$
- $T = \frac{1}{f} = \frac{2\pi}{\omega}$
- Also possible to merge the As and Bs into complex coefs (use Euler's iden.), or use either sin or cos with a phase angle.

ANY periodic waveform $y(t)$ may be expressed as

$$y(t) = A_0 + \sum_{n=1}^{\infty} \left(A_n \cos(n\omega t) + B_n \sin(n\omega t)\right)$$

where

$$A_0 = \frac{1}{T} \int_{-T/2}^{T/2} y(t)\, dt$$

$$A_n = \frac{2}{T} \int_{-T/2}^{T/2} y(t) \cos(n\omega t)\, dt$$

$$B_n = \frac{2}{T} \int_{-T/2}^{T/2} y(t) \sin(n\omega t)\, dt$$