Dr. Zee and the Black Lagoon

Dr. Zee, the fearless world class sailor, was maneuvering his small craft through the dark waters of the Black Lagoon when a sudden gust of wind caught the main sail, and the boat capsized. "Yikes!" shrieked Dr. Zee, "we'll drown..."

"Shh," interrupted his talented assistant, "we must not make too much noise or we'll disturb the 'creature'. We should swim quietly to shore."

"Yes," whispered Dr. Zee, "but in which direction should we swim?"

"Well," answered his assistant, "I noticed that the depth of the water, in meters, is given by the function

$$f(x, y) = 1200 - x^2 - 2y^2,$$

where (x,y) represents the position on the surface with (0,0) being located at the center of the lagoon, and with the y-axis pointing north.

"I've got it!" shouted Dr. Zee, "we'll use the gradient!"

"Not so loud," pleaded his assistant.

"Yes," continued Dr. Zee in a lower voice, "we should swim in the direction where the depth decreases most rapidly, that is, in the opposite direction as the gradient. The gradient is

$$\bigtriangledown f(x, y) = \langle -2x, -4y \rangle,$$

and our current position is (3,4). Thus the value of the gradient at our position is the vector $\langle -6, -16 \rangle$, and consequently, we should swim in the direction of the vector $-\langle -6, -16 \rangle$ or $\langle 6, 16 \rangle$."

"Come with me," concluded Dr. Zee triumphantly, "I'll lead the way!"

"Please, sir, we must not make so much noise," reminded his assistant.