Omega Trek

Dr. Zee (pronounced "Zed"), the famous Canadian astronaut, was in an orbiting space craft comtemplating his upcoming trip to the uncharted planet Omega.

"Hmm," mused Dr. Zee, "Omega is 10 light years from here. We should accelerate at a constant rate for the first 5 light years. If we want to reach a speed of 99% of the speed of light at the halfway point I wonder what our acceleration rate should be?"

"Well," suggested his talented assistant, "perhaps we could use antiderivatives..."

"I've got it!" shrieked Dr. Zee, "We'll use antiderivatives. Let $x(t)$ represent the distance travelled in feet after t seconds and let a represent our constant rate of acceleration. Then

$$x''(t) = a$$

and, taking antiderivative of both sides, we get

$$x'(t) = at + v_0.$$

Since $x'(0) = a(0) + v_0 = v_0$, we see that v_0 represents our initial velocity. Thus, we will set $v_0 = 0$. We now have

$$x'(t) = at.$$

Again, taking antiderivatives, we get

$$x(t) = \frac{1}{2}at^2 + x_0.$$

Since the distance travelled after 0 seconds is certainly 0, we get $0 = x(0) = \frac{1}{3}a \cdot 0^2 + x_0 = x_0$ and

$$x(t) = \frac{1}{3}at^2.$$

Since one light year equals approximately $3.1 \cdot 10^{16}$ feet, the time at which we reach the midpoint of the journey is the solution of the equation

$$x(t) = 5(3.1)10^{16}$$

or

$$\frac{1}{2}at^2 = 5(3.1)10^{16},$$

which is equivalent to

$$t^2 = \frac{10(3.1)10^{16}}{a}$$

or

$$t \approx \frac{(5.6)10^8}{\sqrt{a}}.$$

At the time given above, the velocity is to be 99% of the speed of light or $0.99(9.8)10^8$ feet per second. Thus,

$$x'(\frac{(5.6)10^8}{\sqrt{a}} = 0.99(9.8)10^8 = (0.97)10^8$$

or

$$a \cdot \frac{(5.6)10^8}{\sqrt{a}} = 0.99(9.8)10^8 = (0.97)10^8.$$

Solving for a, we get $a \approx 3.0\frac{\text{ft}}{\text{sec}^2}$. This is equivalent to only $\frac{3.0}{32} = 0.09$g. Substituting $a = 3.0$ into our equation for t we see that the first half of the journey would take

$$t = \frac{(5.6)10^8}{3} = (3.2)10^8 \text{ seconds.}$$

"The trip would take about 20 years," concluded Dr. Zee. "Are there any closer uncharted planets?"