The Hemiplane

Dr. Zee, the famous inventor, was testing his new high performance aircraft, which he called a "hemiplane." He and his capable assistant had constructed this flying machine in his garage.

The cockpit was full of instruments including a meter that showed the slope of the flight path as rise/run. Dr. Zee's flight path was a straight line sloping upward.

"We need to know our rate of climb in meters/sec in order to fill out these government forms," Dr. Zee shouted over the noise of the engine.

"Well," suggested his talented assistant, "we show a ground speed of 90 meters/sec, and the slope indicator shows a two meter rise for each three meters we travel horizontally. If we let x represent our horizontal position and y represent our vertical position, perhaps we could use der..."

"I've got it!" shrieked Dr. Zee, "we'll let x represent our horizontal position, and we'll let y represent our vertical position at time t. Then

$$\frac{dx}{dt} = 90\text{meters/sec}$$

and

$$\frac{dy}{dx} = \text{rise/run} = \frac{2}{3}.$$

In one second, we travel 90 meters horizontally. This means that during one second of travel the horizontal run is 90 meters. The corresponding rise must be 2/3 of 90 meters or 60 meters. Yes, that's it; the rate of ascent is 60 meters/sec."

"Brilliant, Sir!" exclaimed his assistant. "That was most impressive. Since the rate of vertical climb is the derivative $\frac{dy}{dt}$ we have the equations

$$\frac{dy}{dt} = 60$$

and, from the chain rule,

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt}.$$

Thus, $\frac{dy}{dt} = \frac{2}{3}(90) = 60$."