The Hemiplane

Dr. Zee, the famous inventor, was flying over the mountains in a small aircraft shaped like a hemisphere. Dr. Zee had built this plane in his garage, and even though the plane did not use conventional wings but instead relied upon secret electronic devices for lift, he still had to be very careful about ice forming on the exterior. The drive mechanism kept the flat bottom of the craft warm so that no ice could form there, but ice could form on the dome portion. Dr. Zee's faithful and talented assistant was at the controls.

"We seem to be losing altitude," said Dr. Zee's assistant.

"Yikes!" shouted Dr. Zee looking into the rear view mirror at the ruler he had glued to the side of the craft, "we have 1/4 inch of ice on this thing! I wonder what the volume of the ice is? If it is more than 17,000 cubic inches we could crash!"

"Perhaps we could use differentials..." began his talented assistant.

"I've got it!" shouted Dr. Zee, "we'll use differentials to estimate the volume of the ice. The radius of our craft is 100 inches and the volume is given by

$$V = \frac{1}{2} \cdot \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$$

where $r = 100$.

Then

$$dV = \frac{2}{3}\pi 3r^2 dr = 2\pi r^2 dr.$$

If $r = 100$ and $dr = \frac{1}{4}$ then $dV = 2\pi(100)^2(\frac{1}{4}) = 15,708$ cubic inches."

"No problem," concluded Dr. Zee, "half speed ahead, steady as she goes."