ENGR-354 Digital Logic
Introduction

Topics this quarter

Number systems

Boolean algebra
Fundamental logic functions
Optimizing by simplification
More complex logic operations
Arithmetic circuits
Memory
State machines

Number Systems and Their Representations

In this presentation you will learn how numbers are represented in computers.

Numbers in Different Systems

Decimal	Binary	Octal	Hexadecimal
00	00000	00	00
01	00001	01	01
02	00010	02	02
03	00011	03	03
04	00100	04	04
05	00101	05	05
06	00110	06	06
07	00111	07	07
08	01000	10	08
09	01001	11	09
10	01010	12	0A
11	01011	13	0B
12	01100	14	0C
13	01101	15	0D
14	01110	16	0E
15	01111	17	0F
16	10000	20	10
17	10001	21	11
18	10010	22	12

Positional Number Representation

- Decimal
  - $D = d_{n-1}d_{n-2}\ldots d_1 d_0$
  - $V(D) = d_{n-1} \times 10^{n-1} + d_{n-2} \times 10^{n-2} + \ldots + d_1 \times 10^1 + d_0 \times 10^0$
  - Example: $432_{10} = (4x10^2 + 3x10^1 + 2x10^0)_{10}$
- Binary
  - $B = b_{n-1}b_{n-2}\ldots b_1 b_0$
  - $V(B) = b_{n-1} \times 2^{n-1} + b_{n-2} \times 2^{n-2} + \ldots + b_1 \times 2^1 + b_0 \times 2^0$
  - Example: $1101_2 = (1x2^3 + 1x2^2 + 0x2^1 + 1x2^0)_{10}$
- Hexadecimal
  - $H = h_{n-1}h_{n-2}\ldots h_1 h_0$
  - $V(H) = h_{n-1} \times 16^{n-1} + h_{n-2} \times 16^{n-2} + \ldots + h_1 \times 16^1 + h_0 \times 16^0$
  - Example: $6e2f_{16} = (6x16^3 + ex16^2 + 2x16^1 + fx16^0)_{10}$

Conversion: Binary to/from Decimal

- Conversion of binary to decimal
  - $V = b_{n-1} \times 2^{n-1} + b_{n-2} \times 2^{n-2} + \ldots + b_1 \times 2^1 + b_0$
  - $(1101)_2 =$
- Conversion of decimal to binary
  - Use power's of two table or repeated division method.
  - $(857)_{10} =$

Decimal to Binary Conversion Example

Convert $(857)_{10}$

						Remainder	
857	:	2	=	428		1	LSB
428	:	2	=	214		0	
214	:	2	=	107		0	
107	:	2	=	53		1	
53	:	2	=	26		1	
26	:	2	=	13		0	
13	:	2	=	6		1	
6	:	2	=	3		0	
3	:	2	=	1		1	
1	:	2	=	0		1	MSB

Result is $(1101011001)_2$

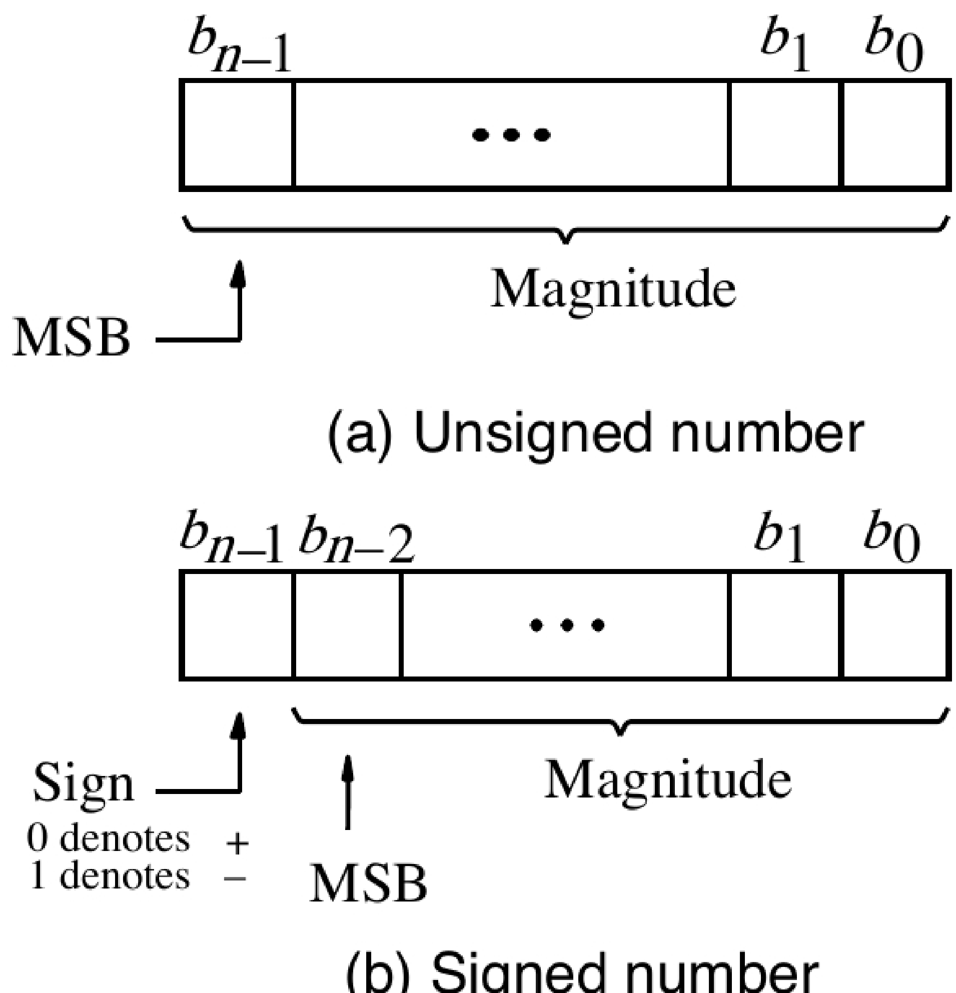
Conversion: Binary to/from Hexadecimal

- Conversion of binary to hexadecimal
  - Groups of four digits, starting with the LSB.
  - $(101001101011101)_2 = (\quad)_{16}$
- Conversion of hexadecimal to binary
  - Reverse of above.
  - $(3f2a)_{16} = (\quad)_2$

Conversion: Decimal to/from Hexadecimal

- Conversion of decimal to hexadecimal
  - Decimal to binary to hexadecimal.
  - $(61)_{10} = (\quad)_2 = (\quad)_{16}$
- Conversion of hexadecimal to decimal
  - Reverse of above.
  - $(3f2a)_{16} = (\quad)_2 = (\quad)_{10}$

Unsigned Vs. Signed Numbers



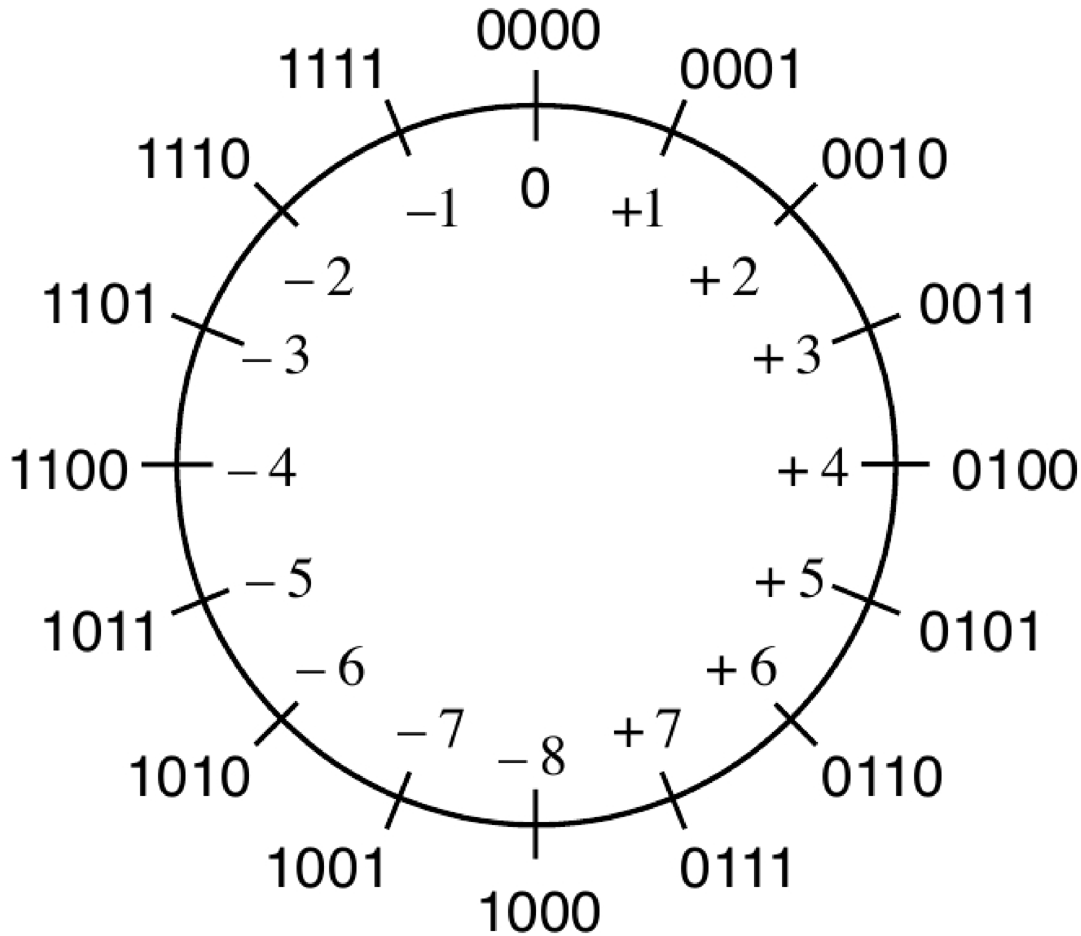
Interpretation of Four-Bit Signed Integers

abcd	Sign and magnitude	1's complement	2's complement
0111	+7	+7	+7
0110	+6	+6	+6
0101	+5	+5	+5
0100	+4	+4	+4
0011	+3	+3	+3
0010	+2	+2	+2
0001	+1	+1	+1
0000	+0	+0	0
1000	-0	-7	-8
1001	-1	-6	-7
1010	-2	-5	-6
1011	-3	-4	-5
1100	-4	-3	-4
1101	-5	-2	-3
1110	-6	-1	-2
1111	-7	-0	-1

2's Complement

- n-bit negative number found by subtracting its positive form from 2^n
  - $K_2 = 2^n - P$
  - $K_2 = K_1 + 1$
- Complement each bit and add 1.
- By far the most common form for signed numbers today.

Graphical Interpretation of Four-bit 2's Complement Numbers



2's Complement Addition Examples

$$\begin{array}{r} (+5) \\ +\ (+2) \\ \hline (+7) \end{array} \qquad \begin{array}{r} 0101 \\ +\ 0010 \\ \hline 0111 \end{array} \qquad\qquad \begin{array}{r} (-5) \\ +\ (+2) \\ \hline (-3) \end{array} \qquad \begin{array}{r} 1011 \\ +\ 0010 \\ \hline 1101 \end{array}$$

$$\begin{array}{r} (+5) \\ +\ (-2) \\ \hline (+3) \end{array} \qquad \begin{array}{r} 0101 \\ +\ 1110 \\ \hline 10011 \end{array} \qquad\qquad \begin{array}{r} (-5) \\ +\ (-2) \\ \hline (-7) \end{array} \qquad \begin{array}{r} 1011 \\ +\ 1110 \\ \hline 11001 \end{array}$$

ignore

ignore

Credit for slide creation: Dr. Nelson

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