ENGR 325 | **Lab 1** | **Walla Walla University**
Winter 2021 | **Calibration** | **Inst. Mark Haun**

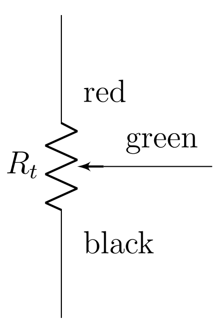
Objectives:

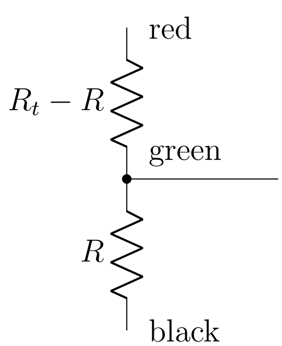
- Practice identifying sources of error in a measurement.
- Learn how calibration can reduce some, but not all, sources of error.
- Review basic circuits principles such as voltage dividers.
- Gain familiarity with lab equipment.

Getting started:
Take inventory of your kit and verify that you have the following parts:

- 1x protractor fixture with three banana sockets (red/green/black)
- 2x black jumper cable with banana plugs
- 1x red jumper cable with banana plugs
- 1x blue jumper cable with banana plugs
- 2x alligator clips (fit over banana plugs)
- Wavetek 220 handheld meter
- battery holder with wire leads
- 4x AA batteries

The protractor fixture is a simple angle-measurement device containing a potentiometer—or "pot"—which is merely a resistor with the addition of a "wiper" or tap point between the ends. As the shaft is turned, the wiper slides back and forth.





A potentiometer is represented schematically by the figure on the left. The color labels refer to the three banana sockets on your fixture. The total resistance between the ends (red and black sockets) is R_t, which does not change. At one limit of shaft rotation, the resistance between red and green will be close to zero and the resistance between black and green will be close to R_t. When the shaft is turned all the way in the other direction, the situation is reversed, with the red-green resistance $\approx R_t$ and the black-green resistance $\approx 0\,\Omega$.

As shown in the figure on the right, the potentiometer can be treated as two resistors in series. As the shaft turns one way, the top resistance grows while the bottom resistance shrinks, and vice versa. The two resistors always add up to R_t. If we call the bottom resistance R, then the top must be $R_t - R$ so that their the sum $(R_t - R) + R = R_t$.

We will use this notation for the remainder of the lab.

(Note: confusingly, the word "potentiometer" can also refer to an entire measurement device or technique. See Section 6.3 in your textbook to learn more.)

Procedure:

Part I: Battery internal resistance

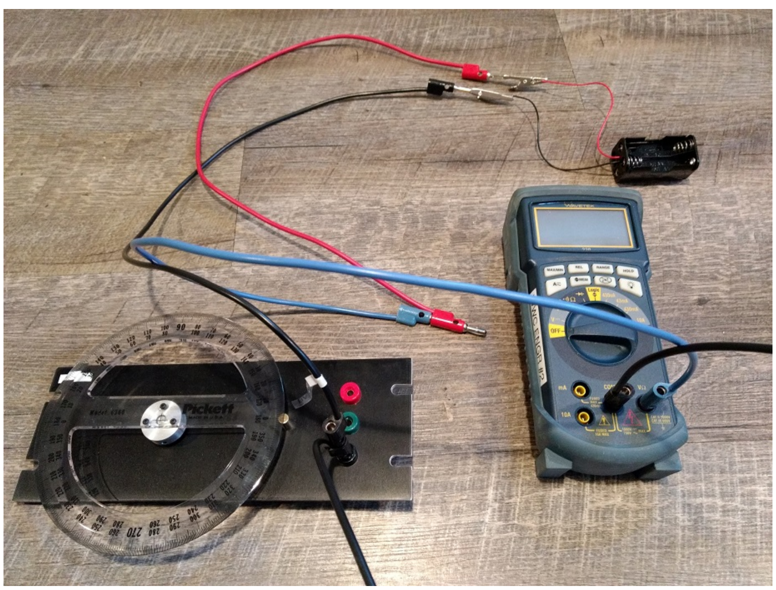
1. Use any two jumper cables to connect the "VΩ" and "COM" sockets (red and black), on the front of the meter, to the endpoints of the potentiometer in the protractor fixture (also red and black). Set the Wavetek meter to measure resistance (look for the Ω symbol on the dial). Measure and record this value, which we call R_t. Disconnect the cables.

2. Select a red and a black jumper cable. Attach two of the alligator clips to one end of these cables. Attach the other end of the red cable to the blue cable. Attach the other end of the black cable to the second black cable. You should now have two long cables:

   alligator → red → blue
   alligator → black → black

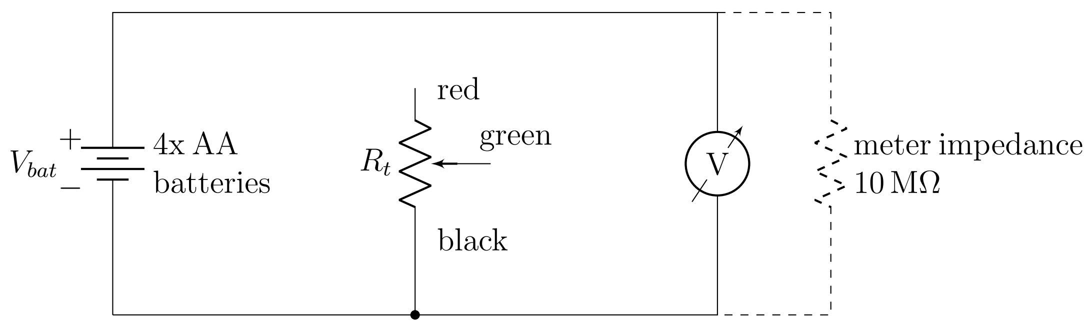
   Plug the far end of the cables (furthest away from the alligator clips) into the Wavetek meter: black to the socket labeled "COM" (which is also black), and blue to the socket labeled "VΩ" (which is colored red).

   Plug the middle of the black cable (the junction between the two black jumpers) into the black socket on the angle-measurement fixture. Soon we will plug the middle of the red/blue cable into the red socket on this fixture, but not yet.



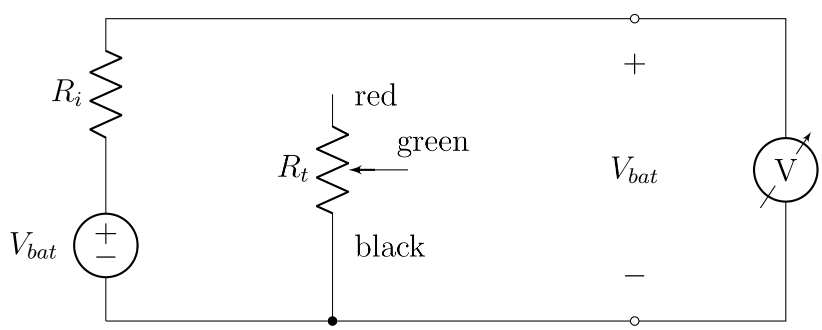
Insert the AA batteries into their holder and clamp the alligator clips onto its wire leads (red to red and black to black). Be careful not to short out the batteries! At this point you should have something like the picture above.

Schematically, what you have built looks like this:



The Wavetek voltmeter is, of course, non-ideal, so it has a finite input resistance. This is shown as the dashed line and 10 MΩ resistor in the diagram. 10 MΩ is thousands of times higher than the other resistances in this circuit—R_t and the battery's internal resistance—so in this lab we can neglect it. It will be omitted from subsequent circuit diagrams.

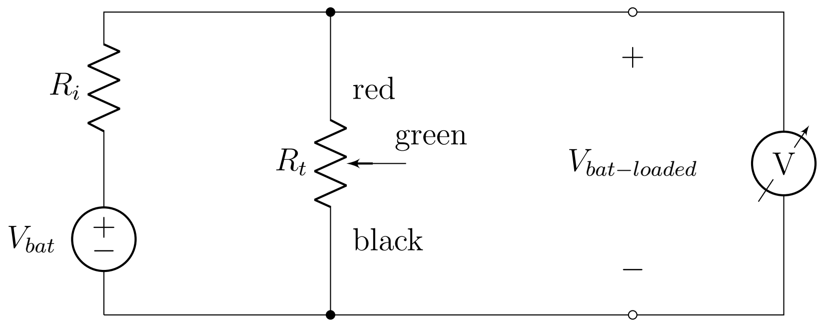
Another non-ideal component in this circuit is the battery pack. Like any real-world power or signal source, it has a Thévenin equivalent circuit consisting of a [perfect] voltage source in series with a resistance. We call this resistance the "internal resistance" of the battery pack, and it reflects the fact that shorting out the pack with a wire does *not* result in near-infinite current flow—fortunately, for anyone who has ever made that mistake! Let's call the battery internal resistance R_i and update the circuit diagram to show it explicitly:



3. Set the Wavetek meter to DC volts mode (the setting closest to "OFF" with the solid and dashed lines) and record the voltage; this is V_{bat}.

   (Why are we ignoring R_i here? Because if we approximate the Wavetek as a perfect voltmeter—which we are—then there is no current flowing in the circuit, and the voltage drop across R_i is zero.)

4. Now plug the junction of red/blue cables into the red socket on the protractor fixture, forming this circuit:



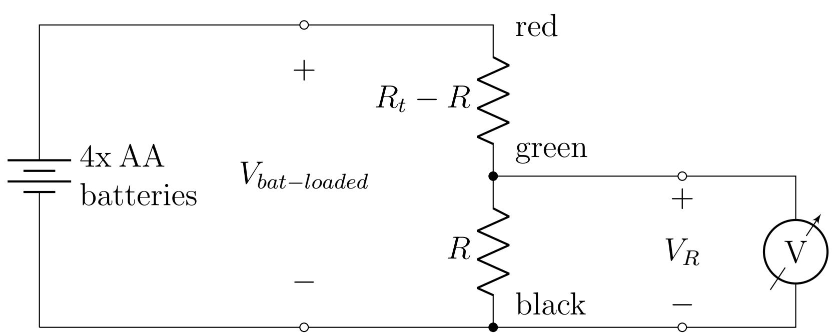
R_t is thousands of times smaller than the voltmeter internal resistance, so the current which flows in the circuit creates a small but non-negligible voltage drop across R_i. In other words, R_i and R_t form a voltage divider, and we are measuring the voltage across R_t. Record this voltage, and call it $V_{bat-loaded}$, the battery voltage under load.

Write down the formula for $V_{bat-loaded}$ as a function of V_{bat}, R_i, and R_t. Then, based on your measurements of R_t (step 1), V_{bat} (step 3), and $V_{bat-loaded}$, calculate the value of R_i.

Part II: Calibration

In this part of the lab, we will treat the battery as a perfect voltage source with a value of $V_{bat-loaded}$. We can do this because the load external to the battery (that is, R_t) does not change over the course of the experiment. Stated another way, there is still a small voltage drop over R_i, but it is constant and already accounted for in $V_{bat-loaded}$.

5. We will shortly reconfigure our circuit to match the following diagram: (For ease of subsequent calculations, the potentiometer has been re-expressed using two series resistors as described in the introduction.)



This is easily accomplished by separating the red and blue cables, leaving the red cable in the red socket on the fixture, and plugging the blue cable into the green socket on the fixture. By moving the blue plug back and forth between the green and red sockets, we can switch between measuring V_R and $V_{bat-loaded}$.

First, with the blue cable still in the red socket (same as the end of Part I), measure and record $V_{bat-loaded}$ one more time. The batteries are slowly draining and measuring the loaded voltage immediately before our calibration run will improve accuracy. At the end of the calibration run, we will measure $V_{bat-loaded}$ again to see how much it changed during the data collection.

6. Now make the change to measuring V_R (blue cable in green socket). Collect angular calibration data every 10° across the full angle range, turning the protractor clockwise from $\theta = 0°$ to about 300° (or wherever it stops) on the outer scale. At each step, read θ as carefully as you can, and record it along with V_R from the voltmeter.

   (Note: It is normal for the meter to show three digits after the decimal point at the lower voltages, and only two digits after the decimal at the higher voltages.)

7. When you are finished taking angle-calibration data, as stated above, switch the blue cable end back to the red socket on the fixture and measure and record $V_{bat-loaded}$ one last time. How much did the loaded battery voltage change while you collected calibration data? If there was a measurable change, how could you use this knowledge to improve the calibration? Decide what to do and explain your reasoning. Whatever you decide, use this to set $V_{bat-loaded}$ in the following calculations.

8. The calibration circuit is a simple voltage divider. Express V_R in terms of R, R_t, and $V_{bat-loaded}$.

   Now solve for R. Enter your data into MATLAB or a spreadsheet and use the formula to convert your V_R versus θ data into a table of R versus θ. (You can use the value of R_t measured in Part I.)

   For an ideal potentiometer, R should be linearly proportional to the shaft angle θ. Examine a plot R vs θ. Does the relationship appear linear? If not, pick the largest angular range you can where the relationship *is* linear, or mostly linear. Denote this range of angles $(\theta_{start}, \theta_{end})$.

   Within this range we would like to model the angle-sensor response with a simple linear equation,

   $$\theta_{predicted} = aR + b$$

   where a is the "gain," or sensitivity, and b is the "offset." Here we consider R as the input (independent) variable and $\theta_{predicted}$ as the output variable. If the model holds, we have an angle transducer where we can easily calculate the angle from the measured resistance, or the measured voltage via the voltage-divider formula.

9. If we assume the response is perfectly linear between θ_{start} and θ_{end}, we can simply draw a line between these points and solve for a and b. Try this by solving the simultaneous linear equations,

   $$\begin{aligned}\theta_{start} &= aR_{\theta_{start}} + b \\ \theta_{end} &= aR_{\theta_{end}} + b\end{aligned}$$

   Write down your linear model using your derived values for a and b.

   For every data point from θ_{start} to θ_{end}, plug your raw calibration data into the model ($\theta_{predicted} = aR+b$) and calculate the difference between the measured θ and $\theta_{predicted}$. (MATLAB or your spreadsheet will be handy here.) The difference should be zero at θ_{start} and θ_{end} because those were the points used to determine the line.

10. A more robust method for determining a and b in the linear model is to use θ_{start}, θ_{end}, and all of the points in between. This can be done with a *least-squares fit*, sometimes known as a *linear regression*, which picks a and b to minimize the square of the residual error. In MATLAB, you can find the least-squares line fit using the commands:

```
q = polyfit(r, theta, 1);
a = q(1);
b = q(2);
```

where `r` and `theta` are the vectors of R and θ. In Excel, see the `LINEST` function: https://support.microsoft.com/en-us/office/linest-function-84d7d0d9-6e50-4101-977a-fa7abf772b6d After computing the new a and b values for the least-squares fit, repeat your calculation of the deviations between each measured θ and $\theta_{predicted}$. What do you observe, compared to the deviations seen in the previous step?

11. Further questions to answer:

    (a) A calibration is only as good as the standard used. In this lab the standard was the protractor, its scale, its mechanical mounting to the potentiometer, and your ability to read the angular scale accurately. What sources of error can you think of in this system? List at least three, and classify each as a random or a systematic error.

    (b) Assuming that we fit using all of the data points in the linear region (as in step 10), do you think that taking more calibration data would improve the quality of the calibration? Why or why not? In what other ways could the calibration be improved?

    (c) Find the measurement resolution in θ due to finite resolution of the voltmeter (that is, the smallest possible increment displayed by the voltmeter). Assume the voltmeter is only showing two digits after the decimal, as it does at higher voltages.

    (d) Which uncertainty do you think dominates in the calibrated angle sensor—the finite angle resolution imposed by the finite voltmeter resolution (previous question), or the likely error caused by imperfect calibration?

    (e) Estimate the total uncertainty for the calibrated angle sensor. Include both the resolution of the voltmeter as well as the calibration error (use the angle deviations in step 10 as a guide).

Guidelines for your lab report (due at the start of lab next week):

To strike the right balance here, imagine you are composing an email summarizing the results of some independent investigation to a boss or coworker. You may assume that your reader already has access to the experimental procedure used, and just focus on summarizing and explaining the results, pointing out significant findings, etc. Grading will be slightly more lenient this week, then we can pick one or two examples to use as models for subsequent weeks.

You should definitely include:

- a description of any procedural issues you ran into during the lab, and how you dealt with them (basically, any difficulties or exceptions to the procedure in this handout)
- your algebra
- key numerical results (e.g. your value for R_i in Part I)
- thoughtful answers to the questions posed throughout the procedure
- a plot of your results in Part II with R on the x-axis and θ on the y-axis, showing
  - your calibration data points

  - the best-fit line using the simple (endpoint-based) solution
  - the best-fit line using the method of least squares

Your plot should comply with the WWU Engineering Graphing Standards. If you are using Excel, be aware that this can take some effort, as most of the default settings are wrong.