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A Mathematical Look at a Medieval Cathedral

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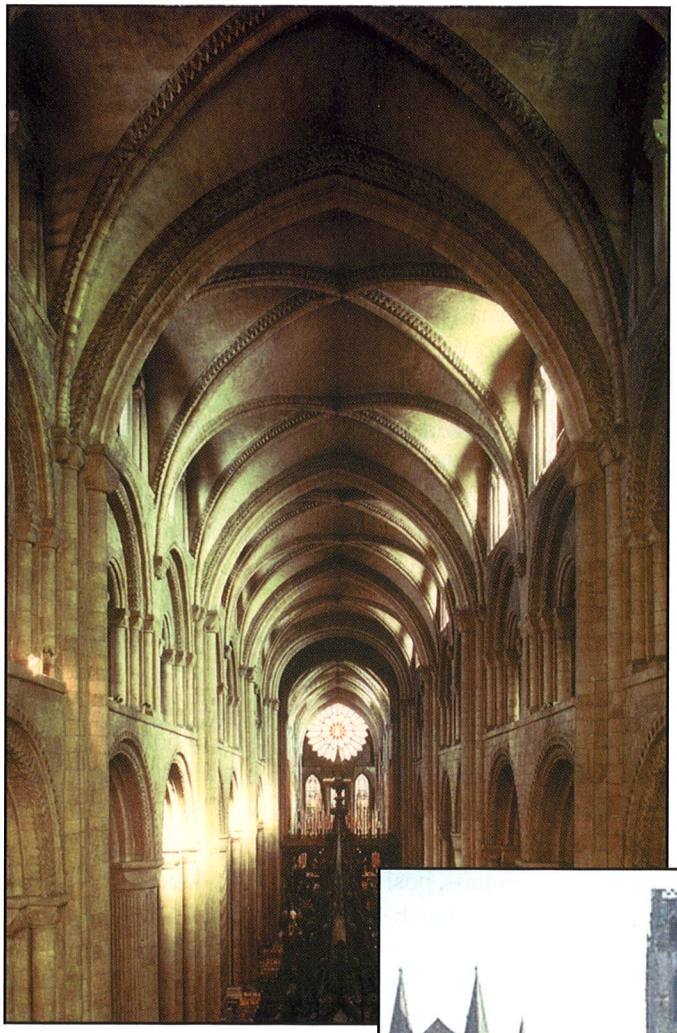


Figure 1. Durham Cathedral: full-length interior view looking toward the liturgical East. Figures 1 and 2, photographs by Peter Coffman.

If you wish to design and build a cathedral, you'd better know some mathematics. The application of mathematics has been central to the design and execution of art and architecture from the Classical era through the Middle Ages and still today. The renown of the Greek prescriptive sculptural instructions, the *Canon of Polykleitos*, attests to this. The celebrated Roman architectural and engineering manual, Vitruvius's *De Architectura*, also emphasized the importance of mathematics in fulfilling the purpose of building. Medieval stonemasonry was itself reverently known as the Art of Geometry. Our focus here will be on the mathematics known and used by medieval stonemasons, in particular in the construction of Durham Cathedral in Northeast England.

One of the main applications of mathematics in medieval architecture was practical geometry. Practical geometry did not concern itself with axioms, deductions, theorems and proofs. Its approach was more empirical and time-tested. Generally, medieval masons including master masons would not have been able to read more abstract or speculative mathematical treatises in Latin, even if they were allowed access to them in the libraries of bishops and monasteries. However, a master mason could adeptly and repeatedly apply a few simple geometric operations and tools, such as the mason's large compass, to produce a myriad of sophisticated designs as attested to by extant late medieval design manuscripts, by full-scale working drawings still etched on some church floors and walls, and by the cathedrals themselves.

The basic tools for design were compasses, dividers,

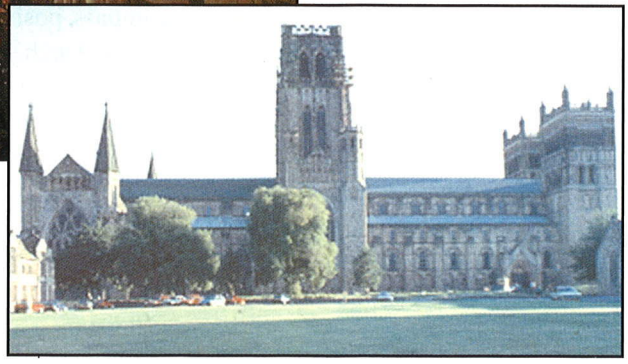
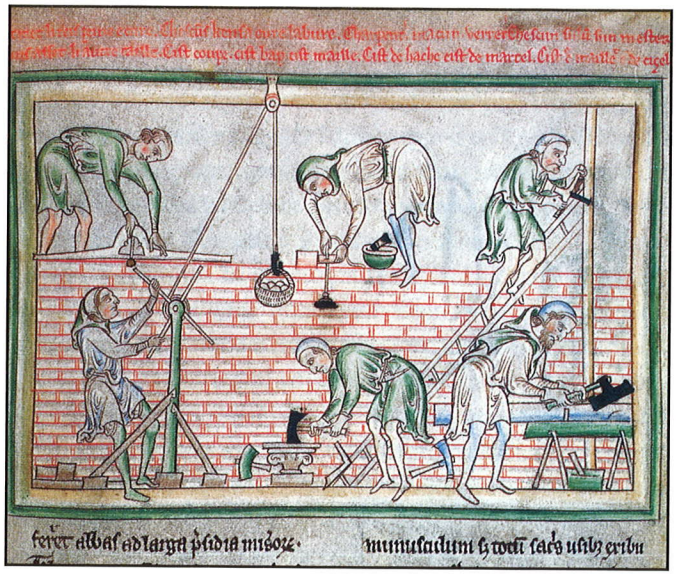
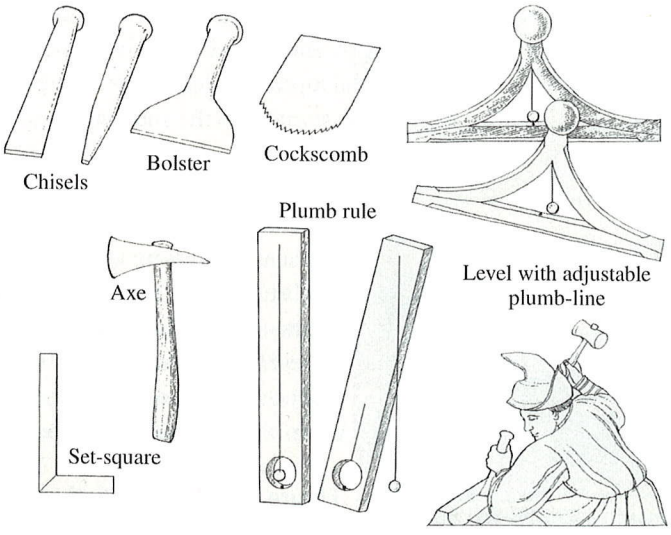


Figure 2. Durham Cathedral exterior, north side.





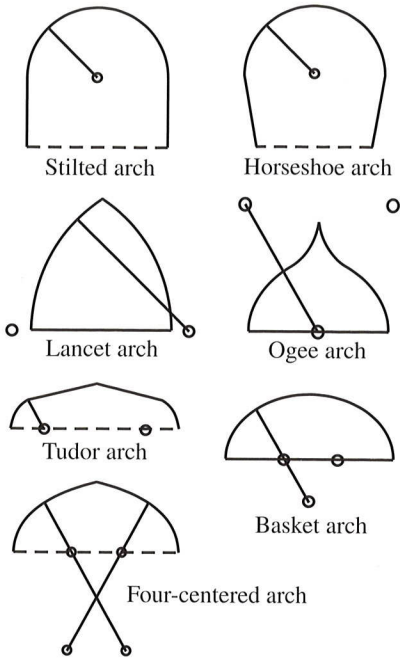


Figure 3. (top left) King Offa and a master mason (holding large compass and set square) instructing stoneworkers at a building site. (middle left) Stonemasons at work on a cathedral wall. This, and the previous image are from *The Book of St. Albans* and reproduced courtesy of the Board of Trinity College Dublin. (bottom left) A stonemason's toolkit, from *Building the Medieval Cathedrals*, Percy Watson. Reprinted with permission from Cambridge University Press.

Figure 4. (above) Various pointed and rounded arch shapes produced with straightedge and compass.

straightedges, rulers, and set squares. See Figure 3. Both small and large compasses and dividers were employed. The large compasses could be up to a meter long. Compasses and dividers were used, of course, for drawing circular arcs, and the latter was also employed to copy or transfer a given length. To implement some larger designs, string and rope could be used to swing out arcs and set out lengths. For drawing straight lines, straightedges, rulers, and set squares would have been employed. However, the central purpose of the set square was, of course, drawing and checking right angles. Simple combinations of compass positions could produce a variety of pointed and rounded arch shapes. See Figure 4.

Hugh of St. Victor (1096–1141) in the twelfth century wrote an influential text, *Practica geometriae*, that made the first distinction between "practical geometry" and "theoretical geometry" in the Latin West. In the history of mathematics, important precursors to this treatise were Euclid's *Elements*, Heron's *Metrica*, the Roman surveyors' *Corpus agrimensorum*, *Ars geometriae* attributed to Boethius, and Gerbert's *Geometria* and the associated anonymous *Geometria incerti auctoris*. Hugh of St. Victor did not actually include the geometry