involved in crafts such as stonemasonry under "practical geometry," only the geometry involved in celestial and terrestrial surveying. Later in the same century, the Spanish philosopher and translator of Arabic, Dominicus Gundissalinus, set forth a schematization of knowledge, *De divisione philosophiae*. Here "practical geometry" was now more broadly defined as involving both the geometric methods of surveying and the crafts. "Theoretical geometry" was basically Euclidean geometry with its axiomatic and deductive proof approach.

We are fortunate to have accounts of consultative building committees that were struck for advice on correcting faults in older buildings or on concerns about new building projects. A central emphasis of these deliberations was that a building should be built with the right proportion and measure for its attractiveness and structural stability. The consultations for Milan Cathedral in 1392 and 1400–1401 are particularly well known and include discussion of whether the building program should continue *ad quadratum* or *ad triangulum* in the elevations. The former expression refers, of course, to using the square and the latter the triangle, and probably, more precisely, the equilateral triangle. In addition to statements from master masons, the assistance of a mathematician named Gabriele Stornaloco was enlisted.

The Architectural Ratios

The ratios of the side to the diagonal of a square, the side to the altitude of an equilateral triangle, and the side to the diagonal of a regular pentagon (later known as the 'golden section'), and their (rational) approximations through ratios of simple whole numbers, are fundamental to medieval architectural design. The side of the square to its diagonal, however, has the strongest pedigree in terms of historical documentation in building manuals, diagrams and illustrations. Examples of (rational) approximations for the side to the diagonal of a square are 5:7, 10:14, 12:17, and 24:34. Apart from such (rational) approximations, even simpler whole number relationships 1:1, 1:2, 2:3 and 3:4 were also employed. Another applied mathematical element involved the use of standard measurement units, such as the English Royal foot (0.3048m), and module lengths. The geometry, ratios and measures worked hand in hand throughout the creative process from the building's design, layout on the ground to the ultimate fabrication and assembly in stone and wood manifesting the completed building.

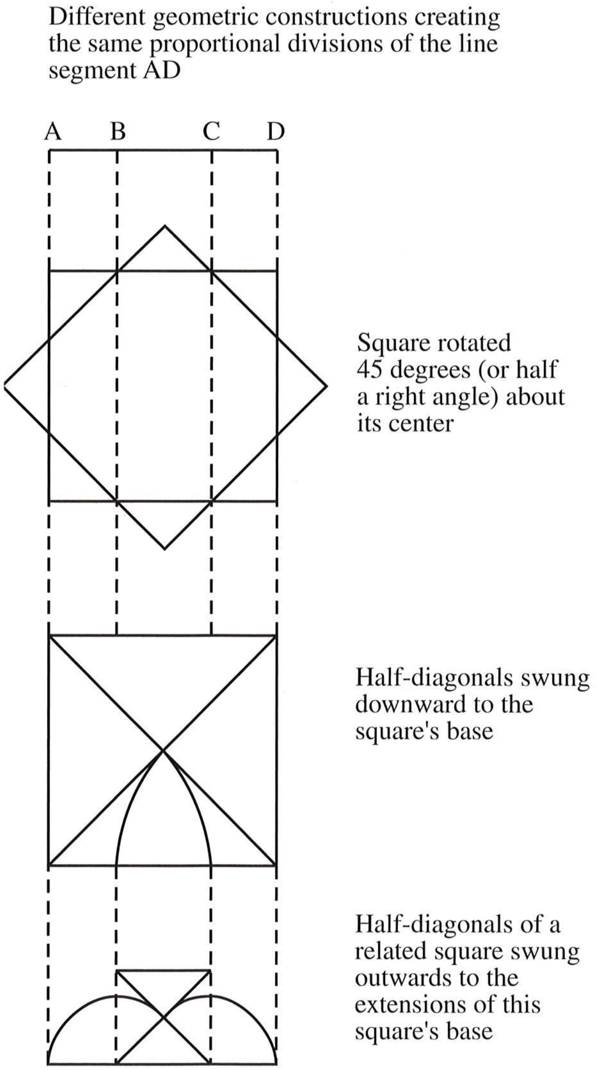

Figure 5. Closely related geometric constructions involving squares, a rotated square, diagonals, and half-diagonals.

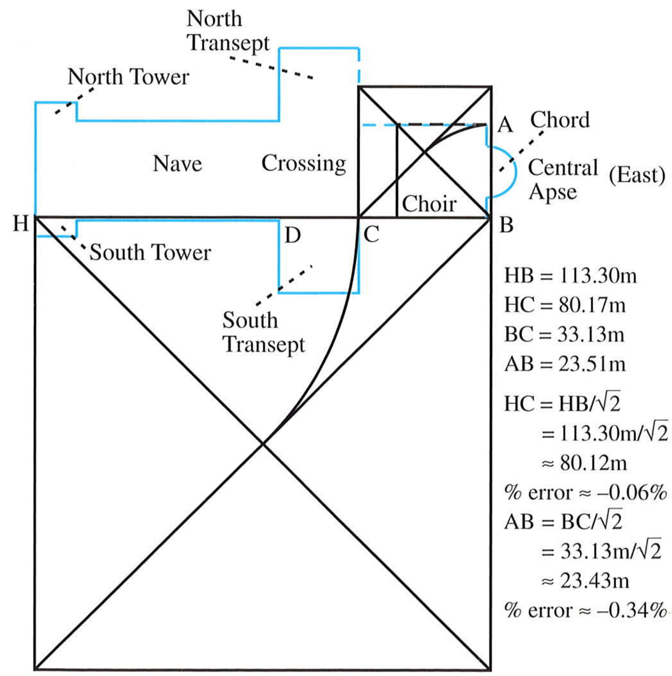

Figure 6. The geometric relationship of the interior length of Durham Cathedral up to the chord of the central apse (HB) and the interior length of the tower, nave and transept (HC), and ii) the interior length (BC) and width (AB) of the choir and east end up to the chord of the central apse. Each of these ratios, *HB* to *HC* and *BC* to *AB*, is as the side to the half-diagonal of a square, i.e., $\sqrt{2} : 1$.

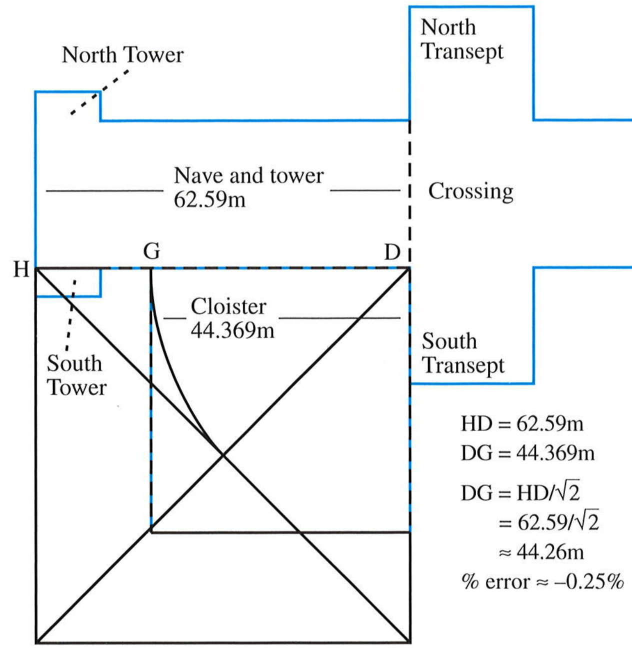

Figure 7. The geometric relationship of the north side of the cloister (DG), and the interior tower and nave (HD) lengths of Durham Cathedral, again we see the side to the half-diagonal of the square.

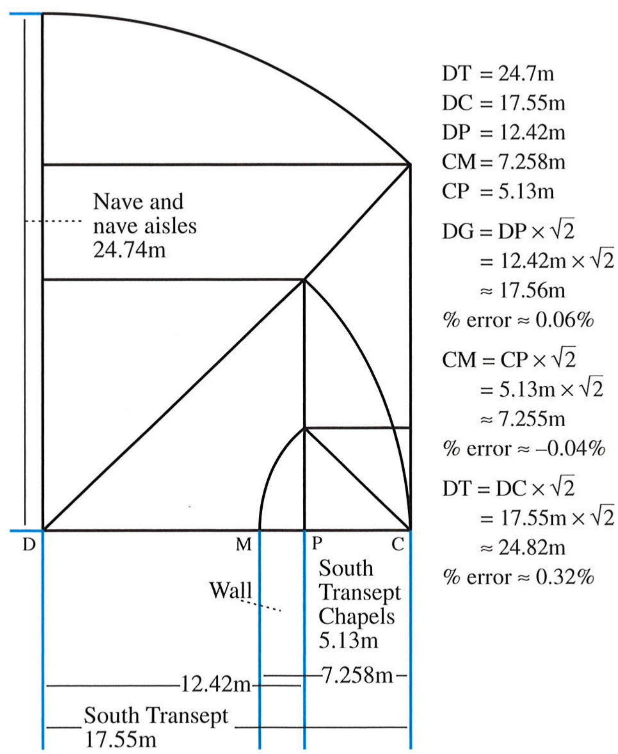

Figure 8. The geometry of the interior ground plan of the south transept, and a relationship with the interior width of the nave and nave aisles of Durham Cathedral, again the ubiquitous $\sqrt{2}:1$ ratio.

To first delve more into the geometry and ratios, one notes, of course, that the square's diagonal to its side, the equilateral triangle's altitude to its side, and the regular pentagon's diagonal to its side are equal to $\sqrt{2}{:}1, \sqrt{3}/2{:}1, (\sqrt{5}+1)/2{:}1$ (golden section), respectively. Specific proportions may be explained by several competing geometric constructions (e.g. Figure 5). However, during the Middle Ages, mathematical knowledge of the irrational numbers, such as $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{5}$, was quite limited. For example, $\sqrt{2}$ was not understood as a non-repeating decimal expansion beginning 1.414213… as commonly understood today. Indeed, even the mathematical terminology $\sqrt{2}$ would have been foreign to medieval masons, in contrast to their familiarity with the geometric maneuvers involving the square's diagonal and its side, and some (rational) approximations involving whole numbers. Nevertheless, there would have been some awareness of the special quality of these ratios and their associated geometric motifs, and even some sense of the implications of the irrationality of these ratios. This is suggested by the Roman architectural forerunner Vitruvius and his discussion of the application of the side and diagonal of a square. He pays great homage to Plato for stating and showing in *Meno* that the square on the diagonal of another square has twice the area of the smaller square. Vitruvius emphasizes the great utility of this result. He notes that this surmounts an arithmetical impossibility (i.e., writing down the square root of two) with a geometric solution. This ascribes to the ratio of the side of the square to its diagonal a special status—it is a profound principle. Its profundity, association with Plato as noted by Vitruvius, and long-standing traditional use may have given a reverence and prestige to this principle during the medieval period.

Mathematical Rediscovery

The rediscovery of the mathematical schema, including the side of the square and its diagonal, employed at a specific church is a challenging problem within architectural history. As an example, Durham Cathedral, an Anglo-Norman Romanesque church, built 1093–1130/1133, in the northeast of England has many mathematical points of interest. Consider the constructional-geometric procedure for the major lengths of the building and the widths of the transepts. A design motif that was common, though not standard, in the large Anglo-Norman Romanesque churches was basically, in terms of interior lengths, that the west tower/nave (HD in Figure 6) to the west tower/nave/crossing/choir up to the chord of the central east-end apse (HB in Figure 6) is in the same ratio as the side of the square to its diagonal or equivalently, the half-diagonal to the side of the square. A slightly different situation appears at Durham Cathedral. The "cut-point" possibly should be the interior east wall of the transept chapels (C in Figure 6), rather than using the interior west wall of the