Figure 9. God as Geometer: Creation as a mathematical act. Vienna, Österreichische Nationalbibliothek, 2554, fol.1 (frontispiece), Bible Moralisée, Reims, c.1250.

transept or the transept piers (D in Figure 6). The length of the choir up to the chord of the central east-end apse (BC in Figure 6) to the width of the choir (AB in Figure 6) are also in the ratio of the side to the half-diagonal of the square. One of the other common larger scale relationships, for Anglo-Norman churches with attached monasteries including Durham, is that the length of the cloister's side adjoining the nave (DG in Figure 7) to the length of the tower and nave (HD in Figure 7) equals the ratio of the side of the square to its diagonal, or equivalently the 'half-diagonal' of the square to its side. The thorough application of the square's side and diagonal also occurred in the ground plan of the south transept and suggests a relationship between the full interior width of the south transept and the interior width of the nave and its north and south aisles [Figure 8]. These relationships are examples of the application of practical or constructive geometry in the design and laying out of Durham Cathedral.

The Deeper Meaning

Practical-geometric methods were thought necessary for a building's structural stability and the visually harmonious and unifying relationship of its individual parts to the whole. From the point of view of the masons, practical geometry allowed for basic designs to be easily repeated and varied. Also, it would have allowed for easy communication to assistants, and for the straightforward teaching of principles and motifs to apprentices. It was understood to be the correct way as indicated in sources from the Roman Vitruvius to the late medieval German master masons. Further, the three geometric ratios already discussed, and related in modern terms to $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{5}$ have a marked simplicity: they derive from the first three regular polygons. This may have stamped them as fundamental design elements or motifs for masonry .

The meaning and symbolism of the cathedral also applied mathematics. The masons' mathematical knowledge applied in building design had a religious significance. In this regard, an important theme in the study of the sacred significance of medieval architecture is the perception of God's Creation as the exemplary handiwork. Part of this theme is Creation as a mathematical act, following biblical passages such as Wisdom 11:21: "Thou madest all things in measure, number and weight." In medieval manuscript illuminations and drawings there are depictions of God creating the cosmos with the aid of a compass or dividers [Figure 9], and sometimes an equal-armed weighing balance as well. God the Creator and orderer implies the divine origin of geometry and measure applied in creating and ordering medieval architecture. The mathematical elements of design had theological import: the masons and builders were co-creators with God, applying mathematics in the creation of the building, a microcosm, to parallel or mirror, God's mathematical creation of the universe, the macrocosm.

The building of a medieval cathedral, and indeed this period's architecture in general, was highly mathematical in conception and execution. Following Abbot Suger's description of the rebuilding of the eastern part of the Abbey Church of St. Denis (1140–1144) and the use there of mathematical instruments, the building of a medieval church was to follow, like the entirety of Creation, the law of Wisdom 11:21. Further, a widespread and standard encyclopedia of the Middle Ages, the *Etymologiae* of St. Isidore of Seville (560–636) includes discussion of weights and measures and Wisdom 11:21, and architecture, and emphasized:

> Take away number in all things and all things perish. Take away computation from the world and everything is wrapped in blind ignorance.

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you say in the introduction that you are not an expert in biology, the idea motivated your nomenclature throughout the thesis. Some immunity manifolds are healthy, some are not. Some even have auto-immune diseases!"

"Just names I gave them to help me describe and understand the mathematical structure. I was doing math, not biology."

"Perhaps, but your scientists were never able to make sense out of the immune system before and there was so much room for rich and beautiful discoveries to grow out of your theories. I just couldn't wait to get to work on it. You must have seen by now how I was able to bring it to play on some questions regarding the improvement of vaccinations and in just a few months some medical researchers working on the disease scleroderma will discover that they can use Serre duality to..."

"But," she interrupted, "you made a mistake. I mean, I made a mistake. I was wrong about equation 3.6. The microchimeric subalgebras don't have to be simply connected, and so definition 3.9 just didn't make any sense and..."

"No," his jaw fell open and he dropped the last little piece of his cookie. "Not equation 3.6! But that was one of my favorite parts. I used that everywhere!"

"Yes, I know. That is how I found you. You see, I caught the mistake just after the *Memoir* was published. It wasn't easy, but I was able to make sure that every copy with the mistake was collected unread and replaced with a corrected version...every copy except the one that was sent separately by private courier here to your house. And that is how I knew..."

"Oh my," he said, stirring his tea vigorously. "Oh my, how careless of me! We will have to do something about that, won't we? Yes, something will have to be done about that." ■

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Art and architecture do need to be seen as separate from mathematics, and traditionally, as mentioned earlier, they went hand in hand as attested in such renowned cases as Polykleitos, the master mason of Durham Cathedral, the consultative committees for Milan Cathedral, Leonardo da Vinci, and Albrecht Dürer. The re-discovery of the mathematics of building a medieval cathedral, stonemasonry, the Art of Geometry, is a worthy part of not only the history of architecture, but also the history of mathematics. ■

For Further Reading

Nicola Coldstream's *Masons and Sculptors*, Medieval Craftsmen series, British Museum and University of Toronto Press (1991), provides a succinct overview of the history and methods of medieval stonemasonry. Eric Fernie's "A Beginner's Guide to the Study of Architectural Proportions and Systems of Length" in *Medieval Architecture and its Intellectual Context: Studies in Honour of Peter Kidson*, (Eds. Paul Crossley and Eric Fernie), Hambledon Press (1990), pp. 229–237, gives important guidelines, with examples, for research on recovering geometry, proportions, and historical measurement units in the design and building of medieval architecture. Wolfgang Weimer's and Gerhard Wetzel's "A Report on Data Analysis of Building Geometry by Computer," *Journal of the Society of Architectural Historians*, vol. 53 (1994), pp. 448–460, shows a particular scientific approach with specially designed computer software to analyze medieval building measurements.