Structural Design
Slender Concrete Column Design In Sway Frames
ACI 318-11

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Introduction
Slender column design for sway frames may be accomplished by one of three methods. These methods are specifically named in the concrete code, ACI 318-11. The three methods are:

1. Second Order Computer Analysis
2. Direct P-Δ Analysis
3. ACI Sway Moment Magnifier Method.

Each of these methods will be described herein. However, several items of note must be stated before we proceed with the explanations for methods 1, 2 and 3.

Load Combinations and Strength Reduction Factors
The ACI code allows a choice for the set of load combinations and strength reduction factors that may be used for design of slender columns in sway frames. The set of load combinations and strength reduction factors must be one of the following sets:

(i) Load combinations and strength reduction factors as given in ACI sections 9.2 and 9.3.

(ii) Load combinations and strength reduction factors as givin in ACI Appendix C.

The β Factors
For load combinations which include ***lateral*** loads the factor, β_{ds}, shall be calculated as follows:

$$\beta_{ds} = \frac{maximum\ factored\ sustained\ shear\ within\ a\ story}{total\ factored\ shear\ in\ the\ story}. \tag{1}$$

For load combinations which include ***gravity loads only*** the factor, β_{dns}, shall be calculated as follows:

$$\beta_{dns} = \frac{factored\ dead\ load\ for\ a\ story}{total\ factored\ load\ in\ the\ story}. \tag{2}$$

The Stability Index, Q

The stability index can be used to determine if a particular story in a frame structure should be called braced or unbraced. The stability index may be calculated as

$$Q = \frac{(\sum P_u)\Delta_0}{V_{us}\ell_u}. \tag{3}$$

Where,
$\sum P_u$ = the sum of factored vertical loads for the story in question
Δ_0 = the 1st order lateral deflection of the top of the story relative to the bottom of the story
ℓ_u = story height
V_{us} = the total shear acting on the given story.

The story is considered ***braced*** if $Q \leq 0.05$. However, the story is considered ***unbraced*** if $Q > 0.05$.

Calculating EI

The formula(s) for calculating EI are specified by ACI. The appropriate formula for calculating EI depends on the context in which the value of EI is to be used. Two cases apply.

Case 1 - Calculating EI to determine P_c. Under this case EI may be calculated as

$$EI = \frac{0.2E_C I_g + E_s I_{se}}{1+\beta_d} \tag{4}$$

,or

$$EI = \frac{0.4E_C I_g}{1+\beta_d}. \tag{5}$$

Case 2 - Calculating EI to determine ψ values, or for use as section property information to be input into a computer analysis program. For this case the value of EI is calculated per ACI section 10.11.1.

$$(EI)_{beams} = \frac{0.35E_C I_g}{1+\beta_d} \tag{6}$$

$$(EI)_{columns} = \frac{0.7E_C I_g}{1+\beta_d} \tag{7}$$

Special Notes

1. In equations (4),(5),(6) and (7) the appropriate value of β_{ds} or β_{dns} is used in place of β_d.
2. Note that for calculating ψ the value of β_{ds} may be taken as zero. Also, when EI is so determined for frames with *temporary* lateral loads, such as wind or seismic, β_{ds} will be zero. See ACI 10.10.4.1 for alternative formulas for calculating EI.

Slenderness Limit for SWAY Frames

It is important to note that for sway frames the slenderness limit is different than the slenderness limit for nonsway frames. According to ACI section 10.10.1, a sway frame column is considered slender when

$$\frac{k\ell_u}{r} > 22. \tag{8}$$

Slender Column Analysis

A slender column analysis is sometimes necessary. The whole point of the analysis is to determine the magnified moments acting on the column due to the applied factored loads *and* slenderness effects. The necessary analysis may be accomplished in one of three ways as previously mentioned above. We now describe, in detail, the procedure for applying each method.

1. Second Order Computer Analysis - Required when $\frac{k\ell_u}{r} > 100$

a. Lateral Load Analysis. In this method the analysis of the frame being designed is performed using a 2nd order computer analysis. A 2nd order computer analysis takes into accout $P - \Delta$ effects automatically. To do this analysis each necessary load combination which includes lateral loads must be used. From the computer analyses the load combination which causes the worst end moments will be used for design. Since the computer is doing a second order analysis the moments calculated will already be magnified. Hence, the values of P_u, M_{u1} and M_{u2} can be read directly from the computer output. This method is the most accurate method of analysis.

b. Special Consideration. None necessary for this method.

c. Gravity Load Stability Check. Per ACI 10.10.2.1, total moment including 2nd order effects in compression members shall not exceed 1.4 times the total moment due to 1st order effects.

2. Direct $P - \Delta$ Analysis

a. Lateral Load Analysis. To determine the magnified moments the following procedure may be used. During this process β_{ds} is generally taken as zero.

i. For the worst load combination that includes lateral loads, a 1st order frame analysis must be done in such a way to obtain nonsway and sway moments. Such moments need to be taken from the computer analysis at the top and bottom joints of the column being designed. The moments will be factored and we call them M_{1ns} and M_{1s} for the bottom joint end moments and M_{2ns} and M_{2s} for the top joint end moments of the column being designed. Be careful to maintain the appropriate signs on these moments when you extract them from the computer analysis.

ii. The stability index, Q, must be calculated per equation (3) and is based on the worst load combination determined during the process of step (i).

iii. Calculate δ_s by using the following formula:

$$\delta_s = \frac{1}{1-Q} \leq 1.5. \tag{9}$$

Note that δ_s is never to be used less than 1.0. If δ_s is greater than 1.5 then analysis method 1 or 3 must be used. Alternatively, the column being designed would need to be redesigned with a bigger cross-section. Note that this limitation on δ_s is exceeded when Q becomes greater than 1/3.

iv. Calculate the magnified moments as

$$M_1 = M_{1ns} + \delta_s M_{1s} \tag{10}$$

$$M_2 = M_{2ns} + \delta_s M_{2s} \tag{11}$$

The larger absolute moment, M_1 or M_2, shall be used for design of the column under consideration. This larger moment is usually called M_2 and it is a factored moment.

b. Special Consideration(need to add effect of little $P\delta$). The maximum moment may occur between the ends of the column being designed. This is ordinarily *not* the case for columns in sway frames. For sway frames the maximum column moment usually occurs at one end of the column. However, under certain conditions this may not be the case. Hence ACI requires that we check for such a condition. Such a condition occurs when

$$\frac{\ell_u}{r} > \frac{35}{\sqrt{\frac{P_u}{f'_c A_g}}}. \tag{12}$$

If equation (12) is true then the column must be designed as a nonsway column based on

$$M_c = \delta_{ns} M_2 \tag{13}$$

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \geq 1 \tag{14}$$

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4 \tag{15}$$

with M_1 and M_2 calculated by using equations (10) and (11) and δ_s calculated per equation (9). The factor β_d (maybe zero) is defined per the load combination under consideration and k is defined for a column in a non-sway frame and will likely need to be determined by using calculated ψ values. Once M_c is calculated the column is designed for P_u and the factored moment M_c.

c. Gravity Load Stability Check. Per ACI 10.10.2.1, total moment including 2nd order effects in compression members shall not exceed 1.4 times the total moment due to 1st order effects.

3. ACI Sway Moment Magnifier Method

a. Lateral Load Analysis. To determine the magnified moments the following procedure may be used. During this process β_{ds} is generally taken as zero.

i. For the worst load combination that includes lateral loads a 1st order frame analysis must be done in such a way to obtain nonsway and sway moments. Such moments need to be taken from the computer analysis at the top and bottom joints of the column being designed. The moments will be factored and we call them M_{1ns} and M_{1s} for the bottom joint end moments and M_{2ns} and M_{2s} for the top joint end moments of the column being designed. Be careful to maintain the appropriate signs on these moments when you extract them from the computer analysis.

ii. Calculate $\sum P_u$ for the story of the column being designed.

iii. Calculate P_c for *each* column in the story of the column being designed. Then calculate $\sum P_c$ for the given story.

iv. Calculate δ_s.

$$\delta_s = \frac{1}{1 - \frac{\sum P_u}{0.75 \sum P_c}} \geq 1.0 \tag{16}$$

v. Calculate the magnified moments per equations (10) and (11), but in this case use δ_s as calculated by equation (16). The larger absolute moment, M_1 or M_2, shall be used for design of the column under consideration. This larger moment is usually called M_2 and it is a factored moment.

b. Special Consideration. Follow the same procedure as given in this handout in section 2(b) except for the following:

- Use δ_s as determined in this section.
- Use M_1 and M_2 as determined in this section.

c. Gravity Load Stability Check. Per ACI 10.10.2.1, total moment including 2nd order effects in compression members shall not exceed 1.4 times the total moment due to 1st order effects.

Conclusion. After having completed one of the three methods of analysis listed above the concrete column cross-section is typically designed using an interaction diagram as is done for short columns. Of course in this case we do the design using the governing values of factored axial load and factored moment as determined from one of our three methods of slender column analysis.

References

(1)American Concrete Institute, *ACI 318-11, Building Code Requirements for Structural Concrete and Commentary*, 2011.

(2)MacGregor and Wight, *Reinforced Concrete - Mechanics and Design*, 5th edition, Prentice-Hall, 2009.

(3)Nilson, Darwin, and Dolan, *Design of Concrete Structures*, 13th edition, McGraw-Hill, 2004.